

EE 435

Lecture 29

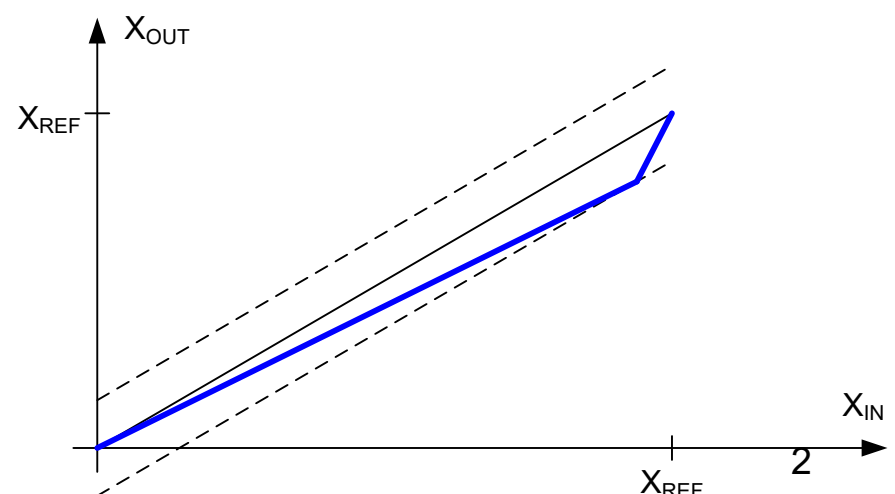
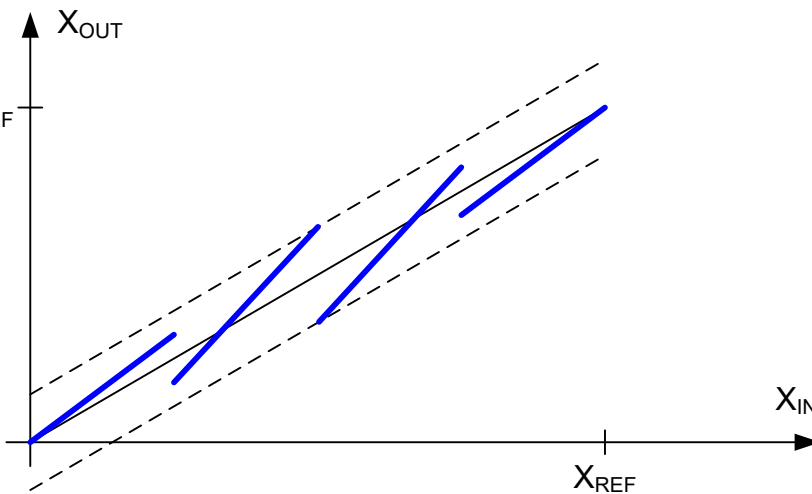
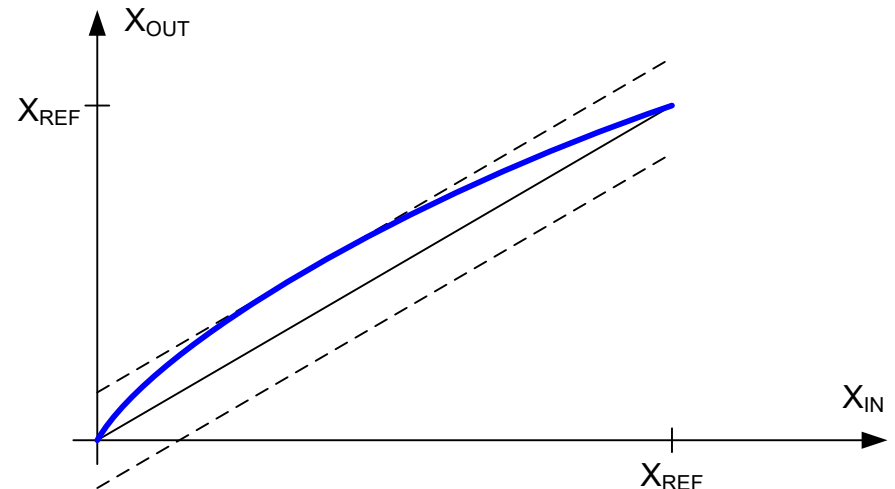
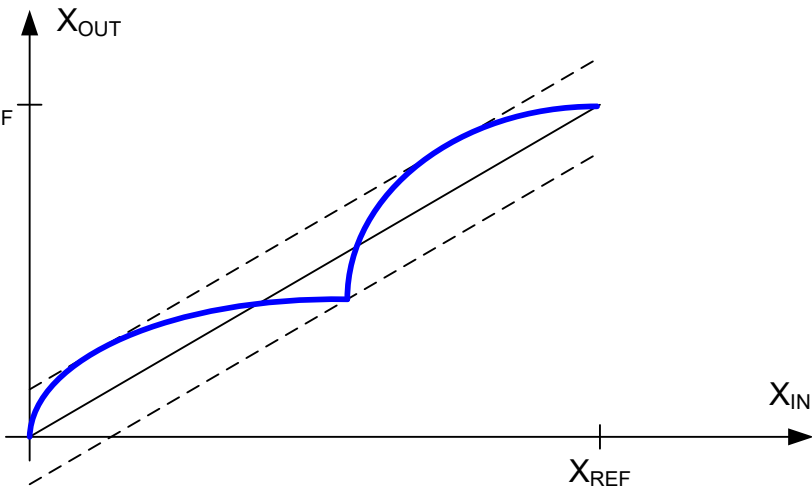
Data Converters

- Spectral Performance
 - Windowing
- Quantization Noise

INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity

Review from last lecture



Distortion Analysis

How are spectral components determined?

By integral

$$A_k = \frac{1}{\omega T} \left(\int_{t_1}^{t_1+T} f(t) e^{-jk\omega t} dt + \int_{t_1}^{t_1+T} f(t) e^{jk\omega t} dt \right)$$

or

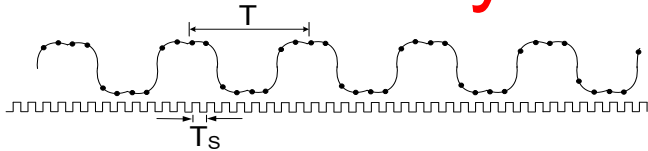
$$a_k = \frac{2}{\omega T} \int_{t_1}^{t_1+T} f(t) \sin(kt\omega) dt \quad b_k = \frac{2}{\omega T} \int_{t_1}^{t_1+T} f(t) \cos(kt\omega) dt$$

Integral is very time consuming, particularly if large number of components are required

By DFT (with some restrictions that will be discussed)

By FFT (special computational method for obtaining DFT)

Why is this a Key Theorem?



$$x(t) = A_0 + \sum_{k=1}^{h-1} A_k \sin(k\omega t + \theta_k)$$

THEOREM: Consider a periodic signal with period $T=1/f$ and sampling period $T_s=1/f_s$. If N_p is an integer, $x(t)$ is band limited to f_{MAX} , and $f_s > 2f_{max}$, then

$$|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h-1$$

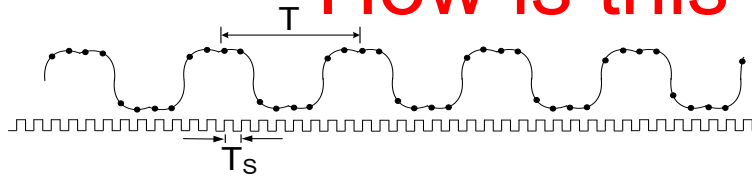
and $X(k) = 0$ for all k not defined above

where $\langle X(k) \rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_s) \rangle_{k=0}^{N-1}$

$\langle A_k \rangle$ are the Fourier Series Coefficients, N =number of samples, N_p is the number of periods, and $h = \text{Int} \left(\frac{f_{MAX}}{f} - \frac{1}{N_p} \right)$

- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem
- If “signal” is output of a system (e.g. ADC or DAC), f_{MAX} is independent of f

How is this theorem abused?



$$x(t) = A_0 + \sum_{k=1}^{h-1} A_k \sin(k\omega t + \theta_k)$$

THEOREM: Consider a periodic signal with period $T=1/f$ and sampling period $T_s=1/f_s$. If N_p is an integer, $x(t)$ is band limited to f_{MAX} , and $f_s > 2f_{max}$, then

$$|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h - 1$$

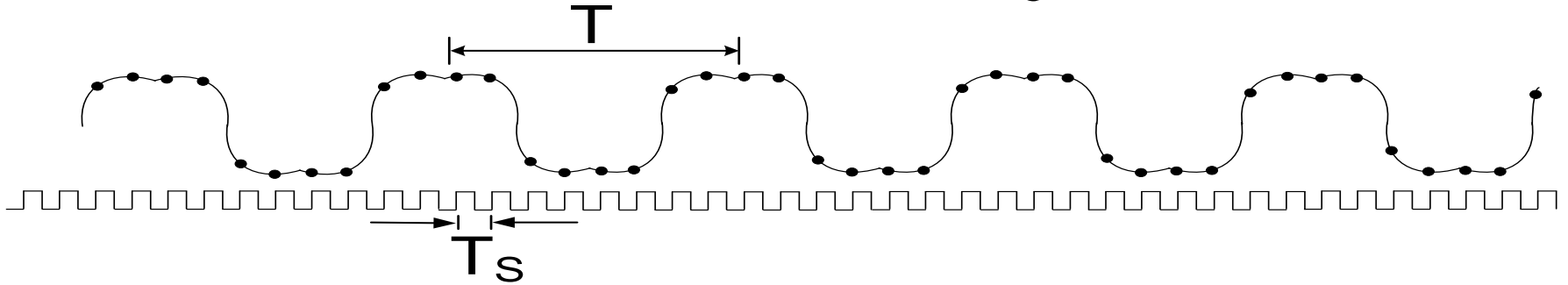
and $X(k) = 0$ for all k not defined above

where $\langle X(k) \rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_s) \rangle_{k=0}^{N-1}$

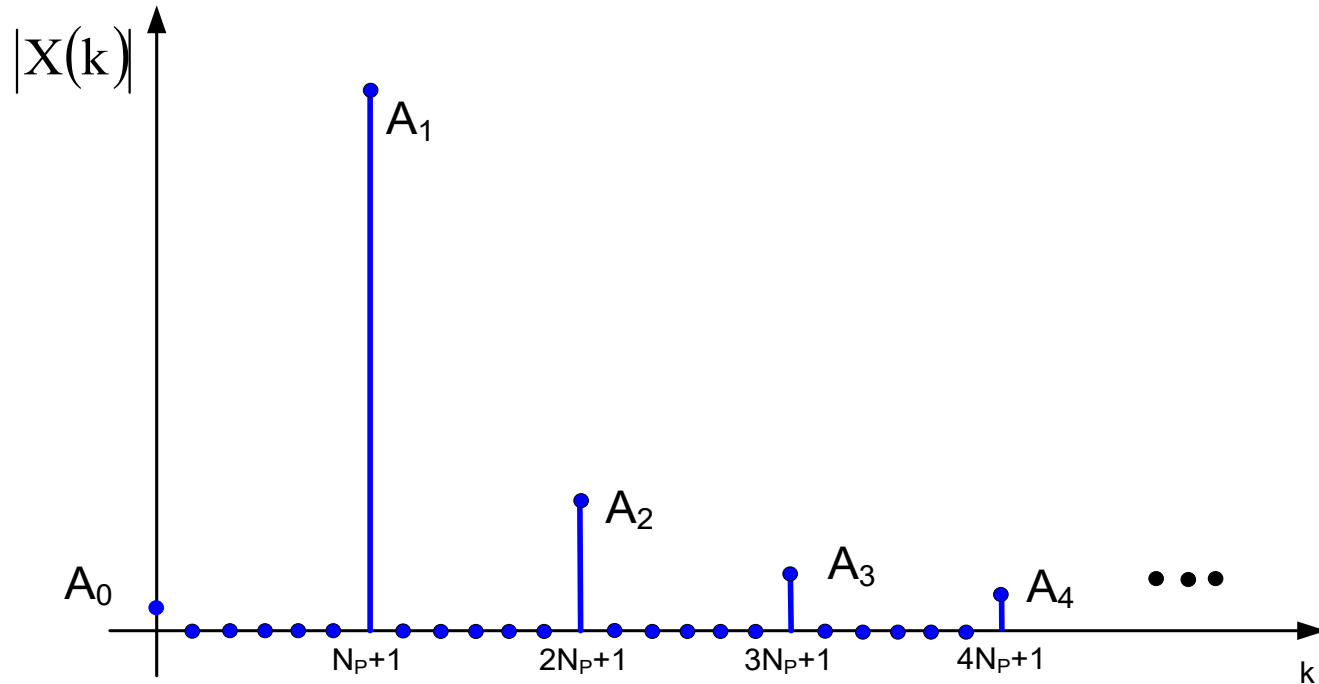
$\langle A_k \rangle$ are the Fourier Series Coefficients, N =number of samples, N_p is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_p}\right)$

- Much evidence of engineers attempting to use the theorem when N_p is not an integer
- Challenging to have N_p an integer in practical applications
- Dramatic errors can result if there are not exactly an integer number of periods in the sampling window

Distortion Analysis



If the hypothesis of the theorem are satisfied, we thus have




• • • • • Review from last lecture • • • • •

Considerations for Spectral Characterization

- Tool Validation
- • DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

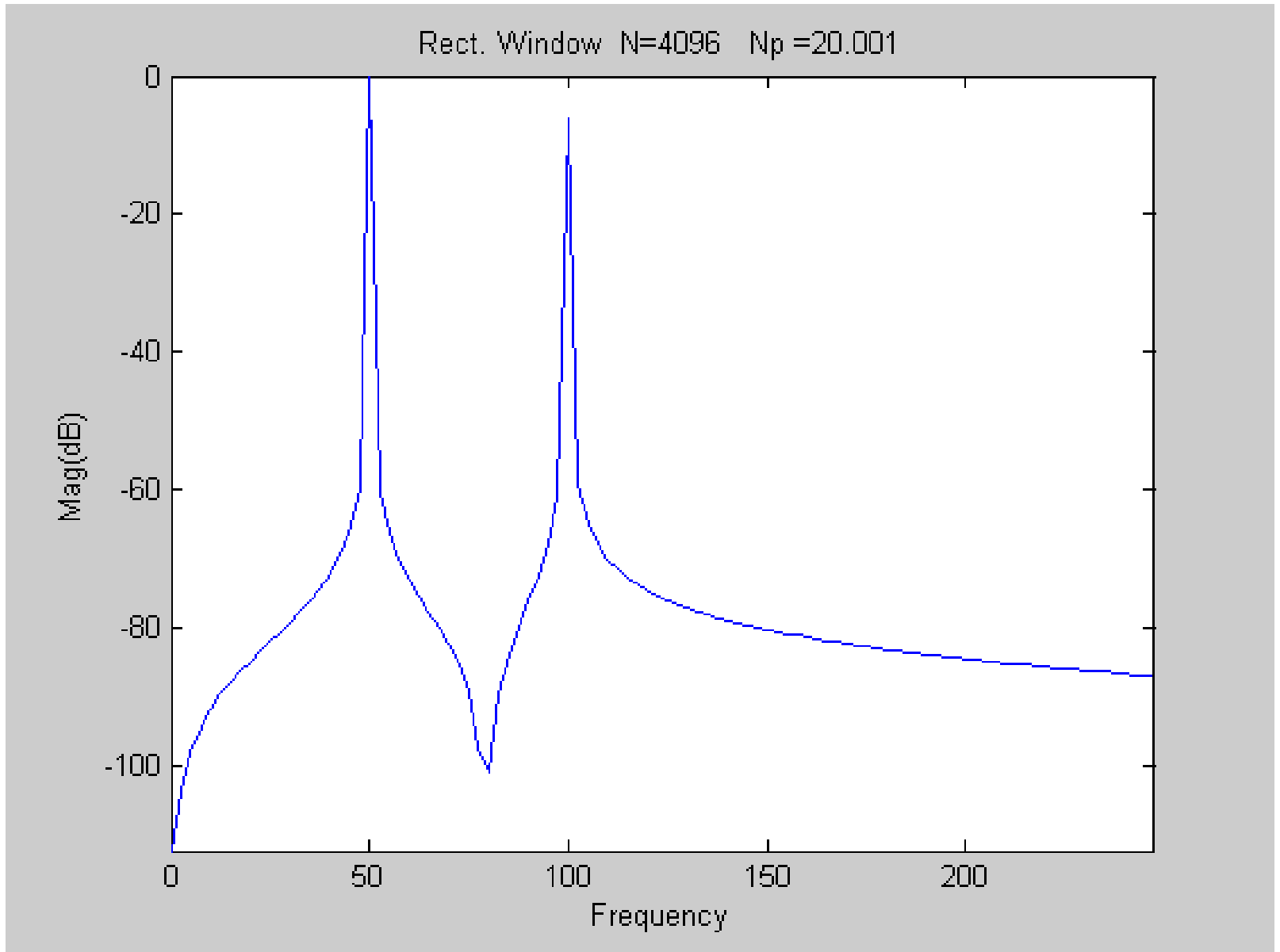
• • • • • Review from last lecture • • • • •

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
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Review from last lecture

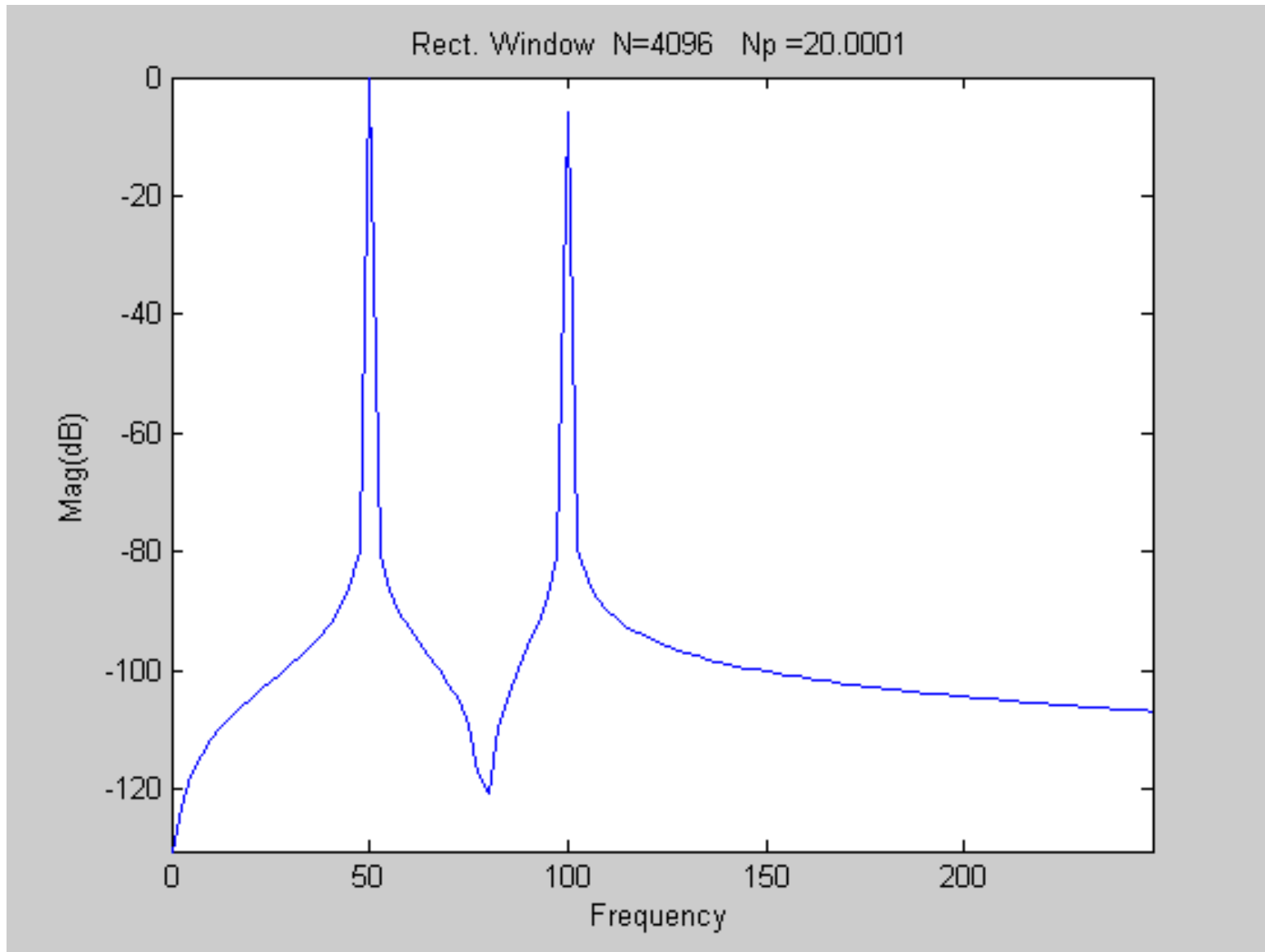
Spectral Response with Non-coherent Sampling



(zoomed in around fundamental)

• • • • • Review from last lecture • • • • •

Spectral Response with Non-coherent sampling



(zoomed in around fundamental)

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
 - NP is an integer
 - Band-limited excitation
- Windowing

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

 2.
$$N > \frac{2f_{\max}}{f_{\text{SIGNAL}}} N_P$$

Example $N < \frac{2f_{\max}}{f_{\text{SIGNAL}}} N_P$ (Not meeting Nyquist sampling rate requirement)

If $f_{\text{SIG}}=50\text{Hz}$

and $N_P=20$ $N=512$

$$N < \frac{2f_{\max}}{f_{\text{SIGNAL}}} N_P \quad \longrightarrow \quad f_{\max} < 640\text{Hz}$$

Example $N < \frac{2f_{\max}}{f_{\text{SIGNAL}}} N_P$ (Not meeting Nyquist sampling rate requirement)

Consider $N_P=20$ $N=512$

If $f_{\text{SIG}}=50\text{Hz}$ but an additional input at 700Hz is present

$$N_P = \frac{NT_s}{T} \quad \leftrightarrow \quad f_{\text{SAMP}} = f_{\text{SIGNAL}} \frac{N}{N_P} \quad f_{\text{SAMP}} = 1.280\text{KHz}$$

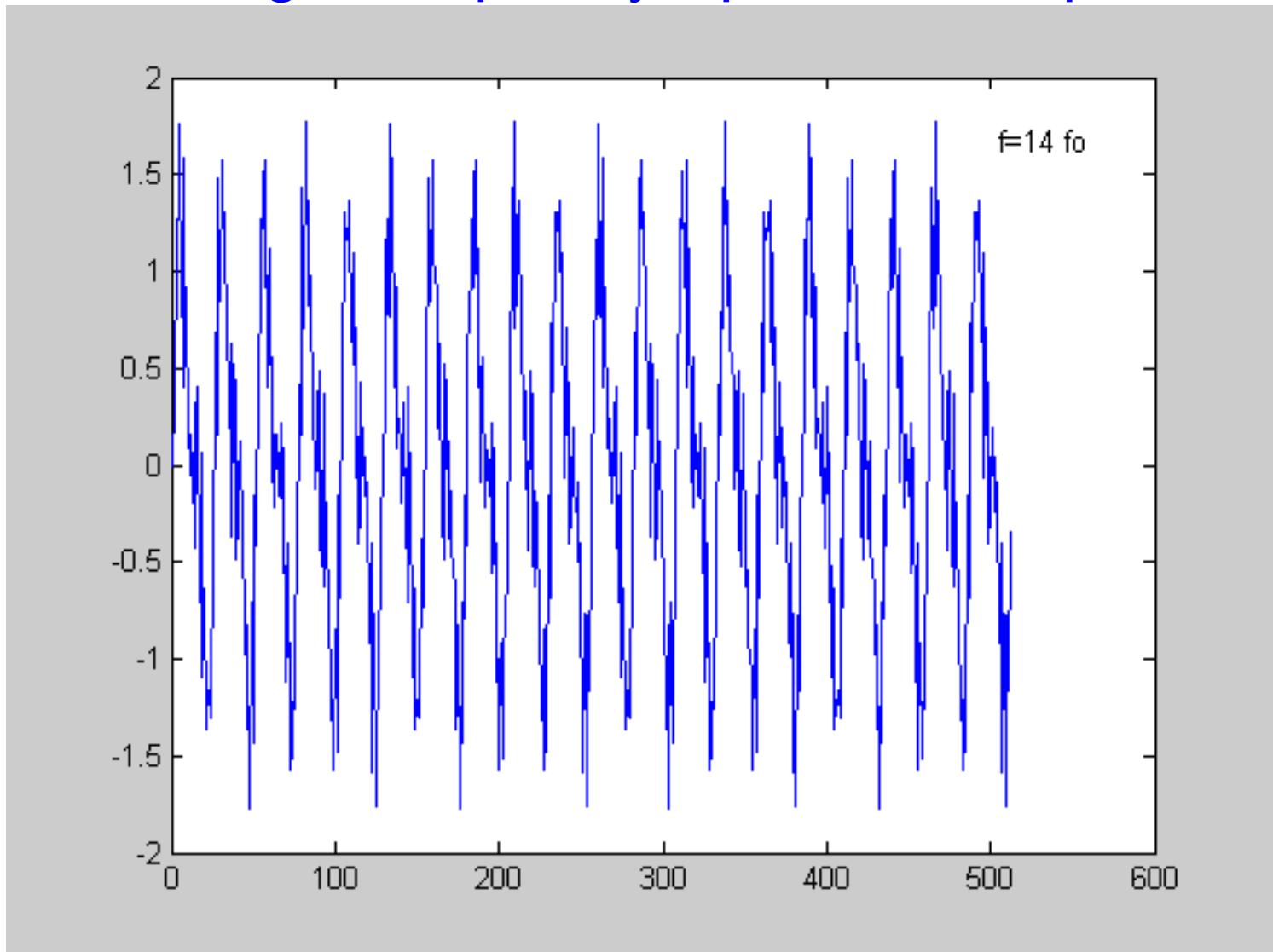
$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t) + 0.5 \sin(14\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

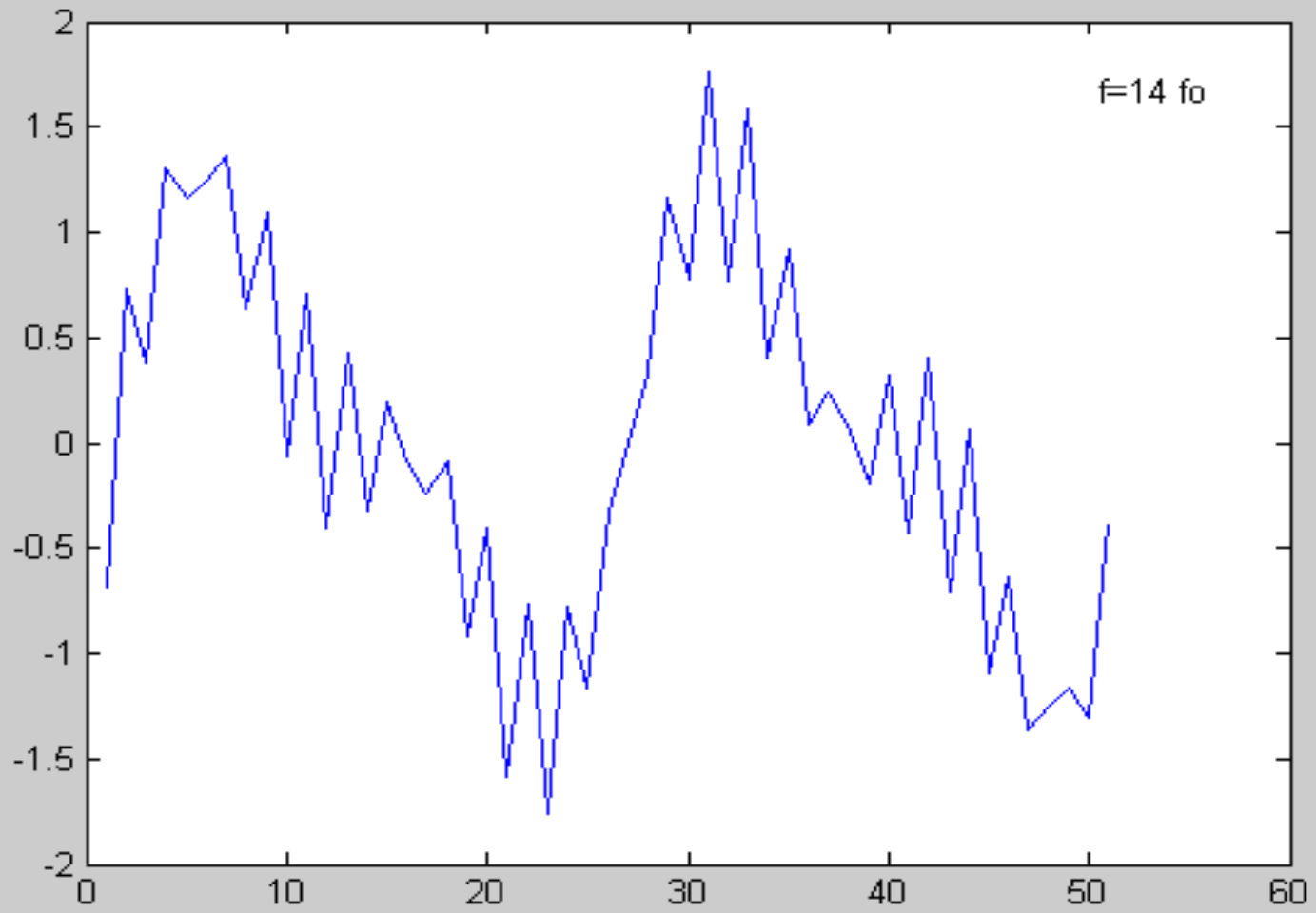
(i.e. the component at 700 Hz which violates the band limit requirement – Nyquist rate for the 700 Hz input is 1.4KHz)

Recall $20\log_{10}(0.5)=-6.0205999$

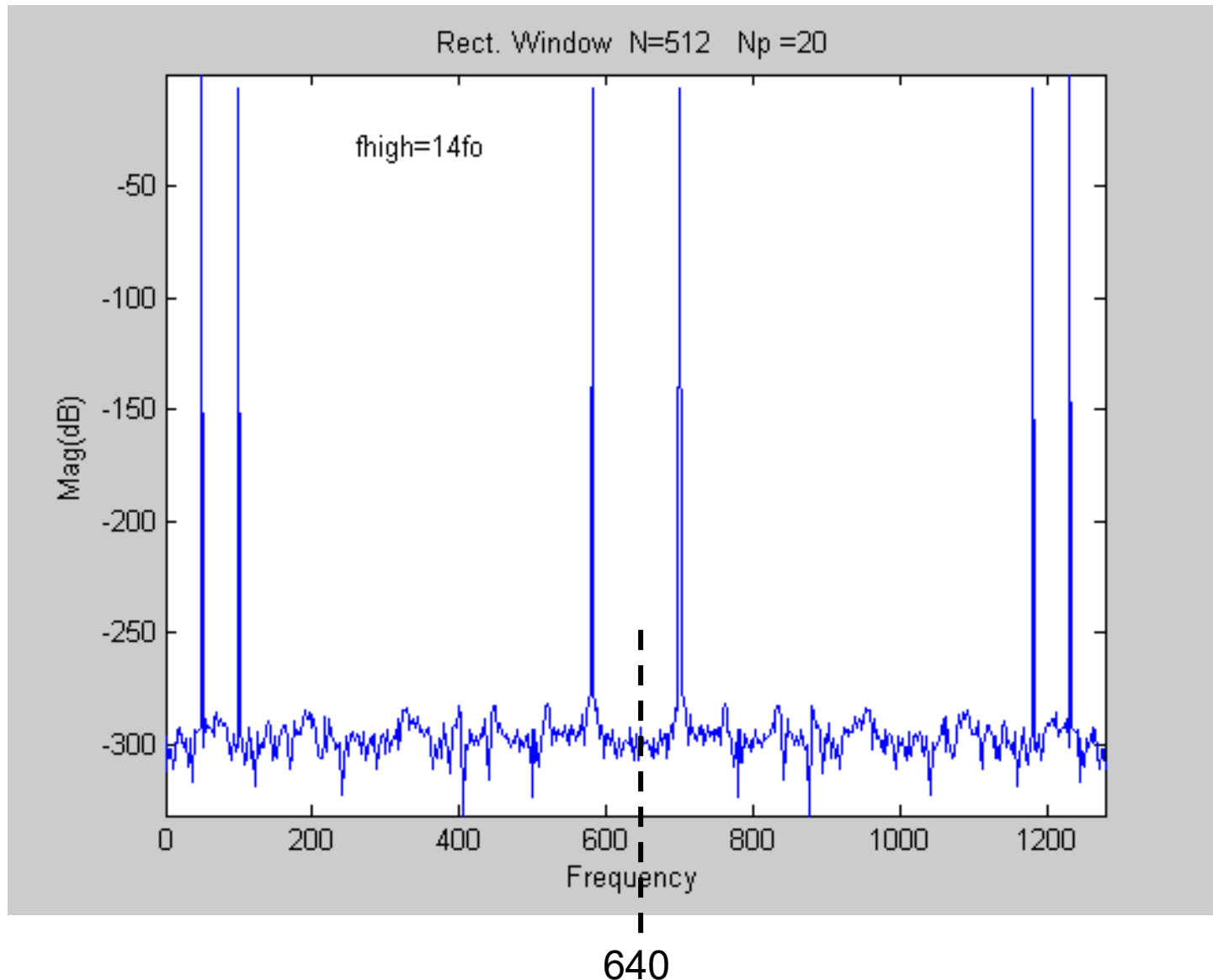
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

$$f_{\text{high}} = 14f_0$$

Columns 1 through 7

-296.9507 -311.9710 -302.4715 -302.1545 -310.8392 -304.5465 -293.9310

Columns 8 through 14

-299.0778 -292.3045 -297.0529 -301.4639 -297.3332 -309.6947 -308.2308

Columns 15 through 21

-297.3710 -316.5113 -293.5661 -294.4045 -293.6881 -292.6872 -0.0000

Columns 22 through 28

-301.6889 -288.4812 -292.5621 -292.5853 -294.1383 -296.4034 -289.5216

Columns 29 through 35

-285.9204 -292.1676 -289.0633 -292.1318 -290.6342 -293.2538 -296.8434

Effects of High-Frequency Spectral Components

$$f_{\text{high}} = 14f_0$$

Columns 36 through 42

-301.7087 -307.2119 -295.1726 -303.4403 -301.6427 -6.0206 -295.3018

Columns 43 through 49

-298.9215 -309.4829 -306.7363 -293.0808 -300.0882 -306.5530 -302.9962

Columns 50 through 56

-318.4706 -294.8956 -304.4663 -300.8919 -298.7732 -301.2474 -293.3188

Effects of High-Frequency Spectral Components

Aliased components at

$$f_{alias} = f_{sample} - f$$

$$f_{alias} = 25.6f_{sig} - 14f_{sig} = 11.6f_{sig}$$

$$\text{thus position in sequence} = 1 + N_p \frac{f_{alias}}{f_{sig}} = 1 + 20 \cdot 11.6 = 233$$

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238

-273.9840 -6.0206 -274.2295 -284.4608 -283.5228 -297.6724 -291.7545

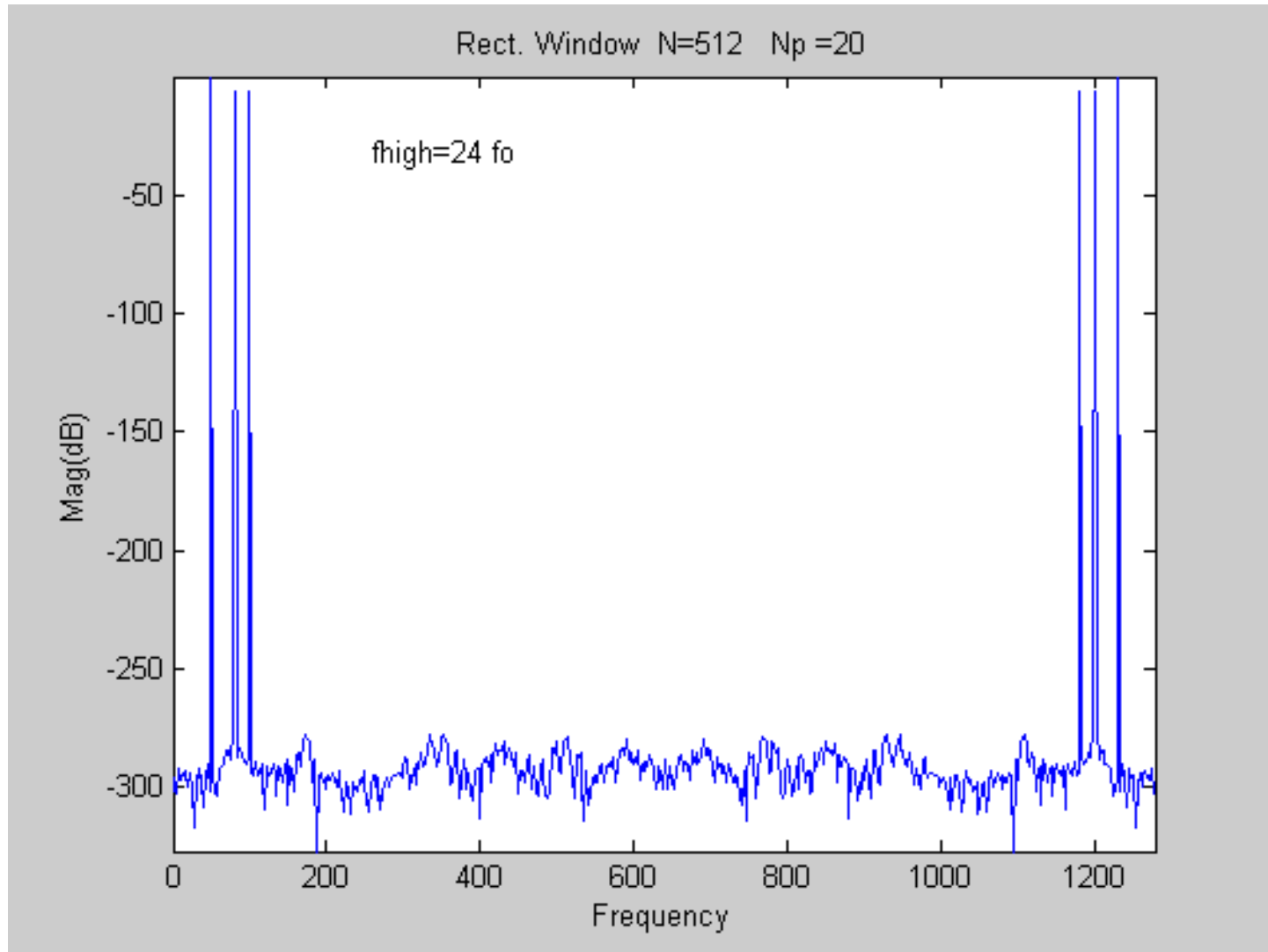
Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

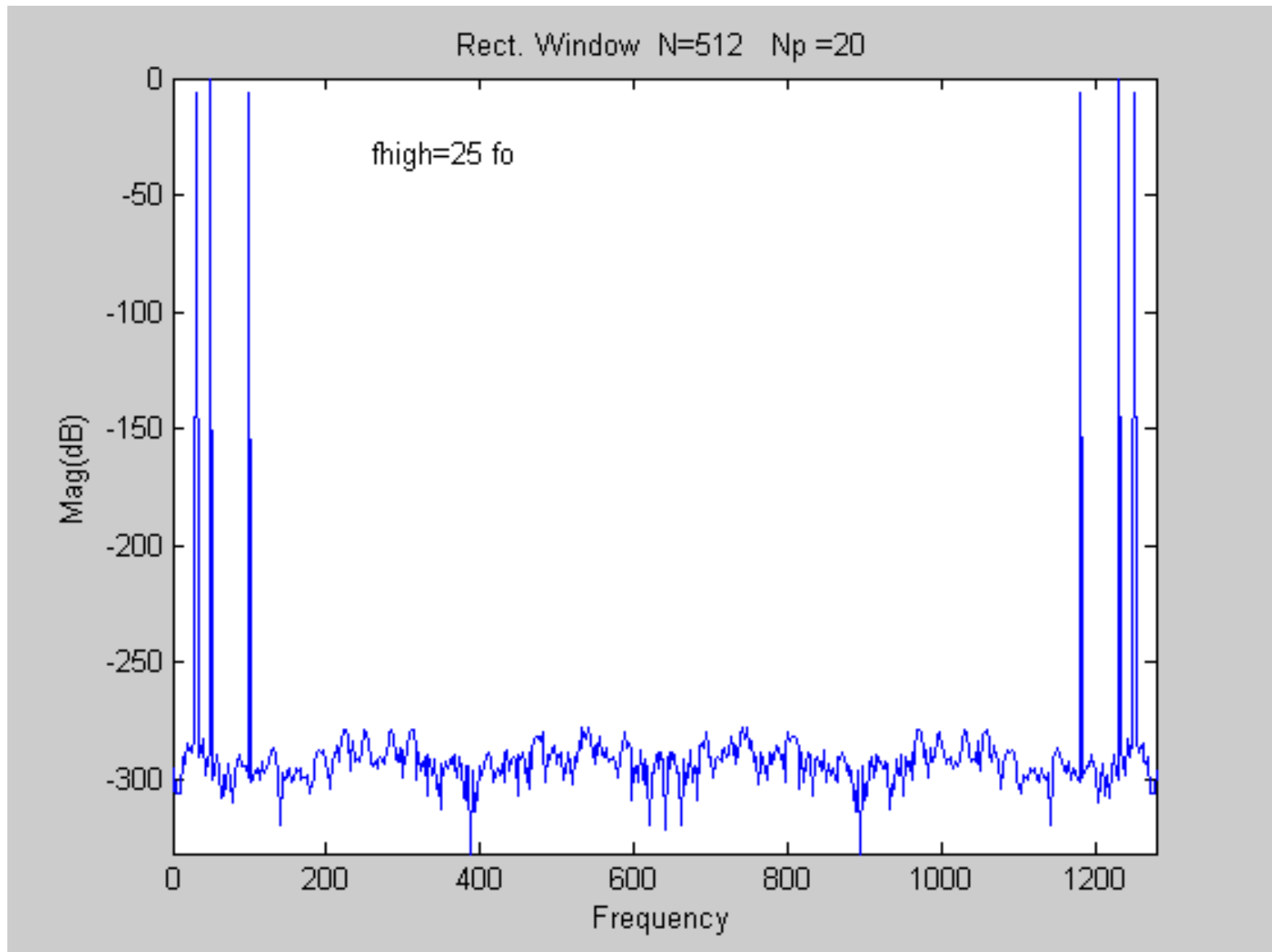
Columns 246 through 252

-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956

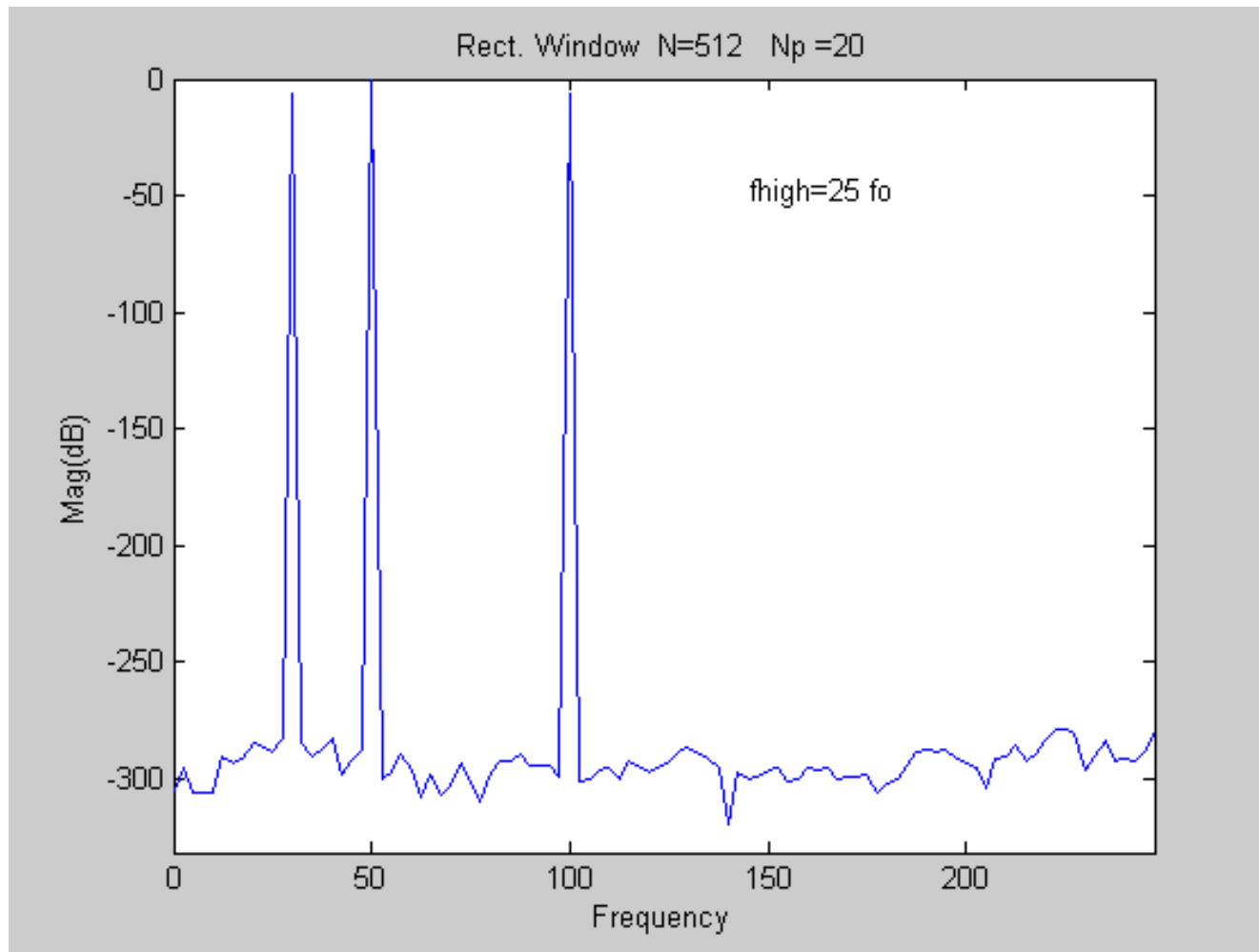
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

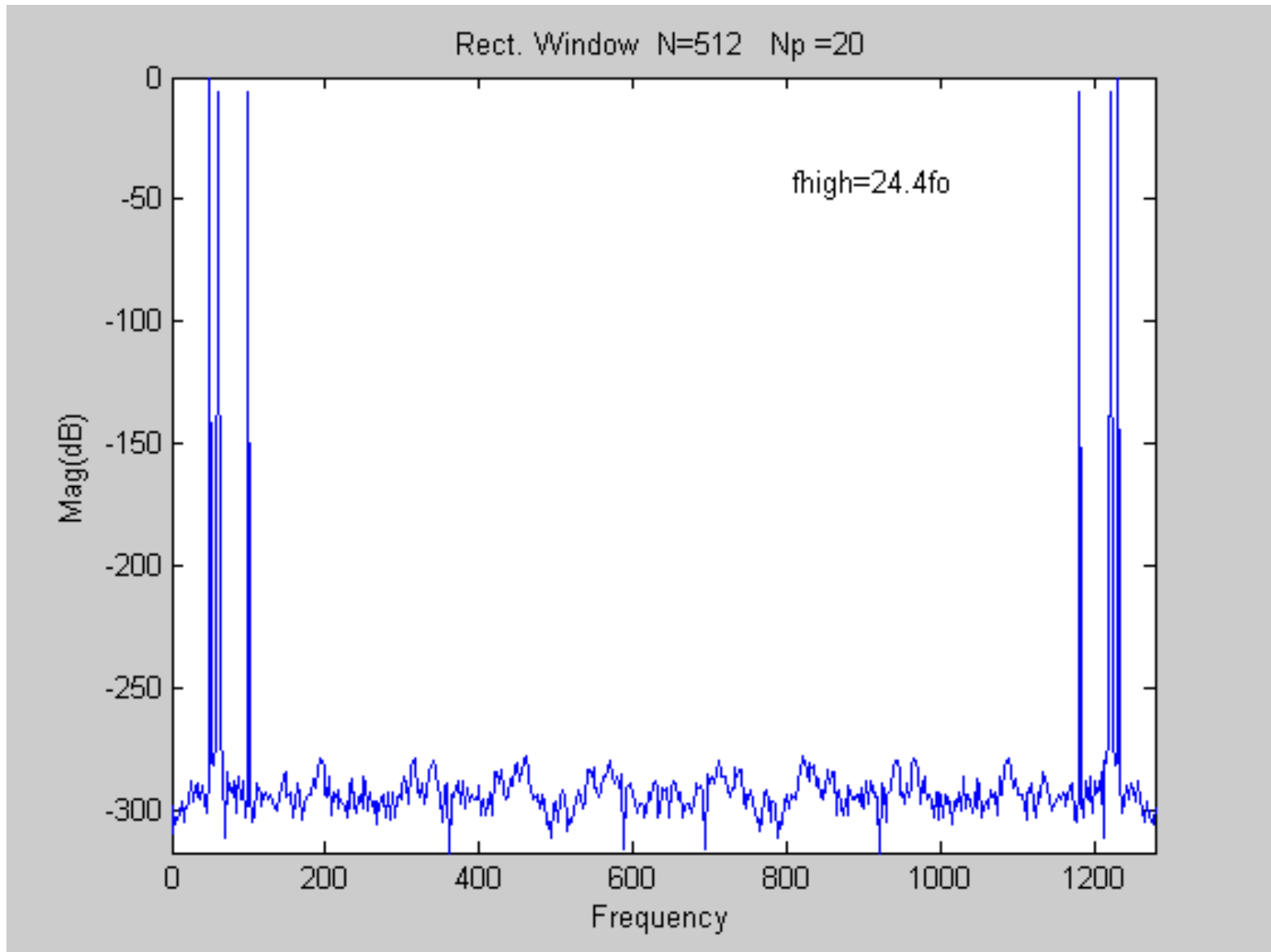


Effects of High-Frequency Spectral Components

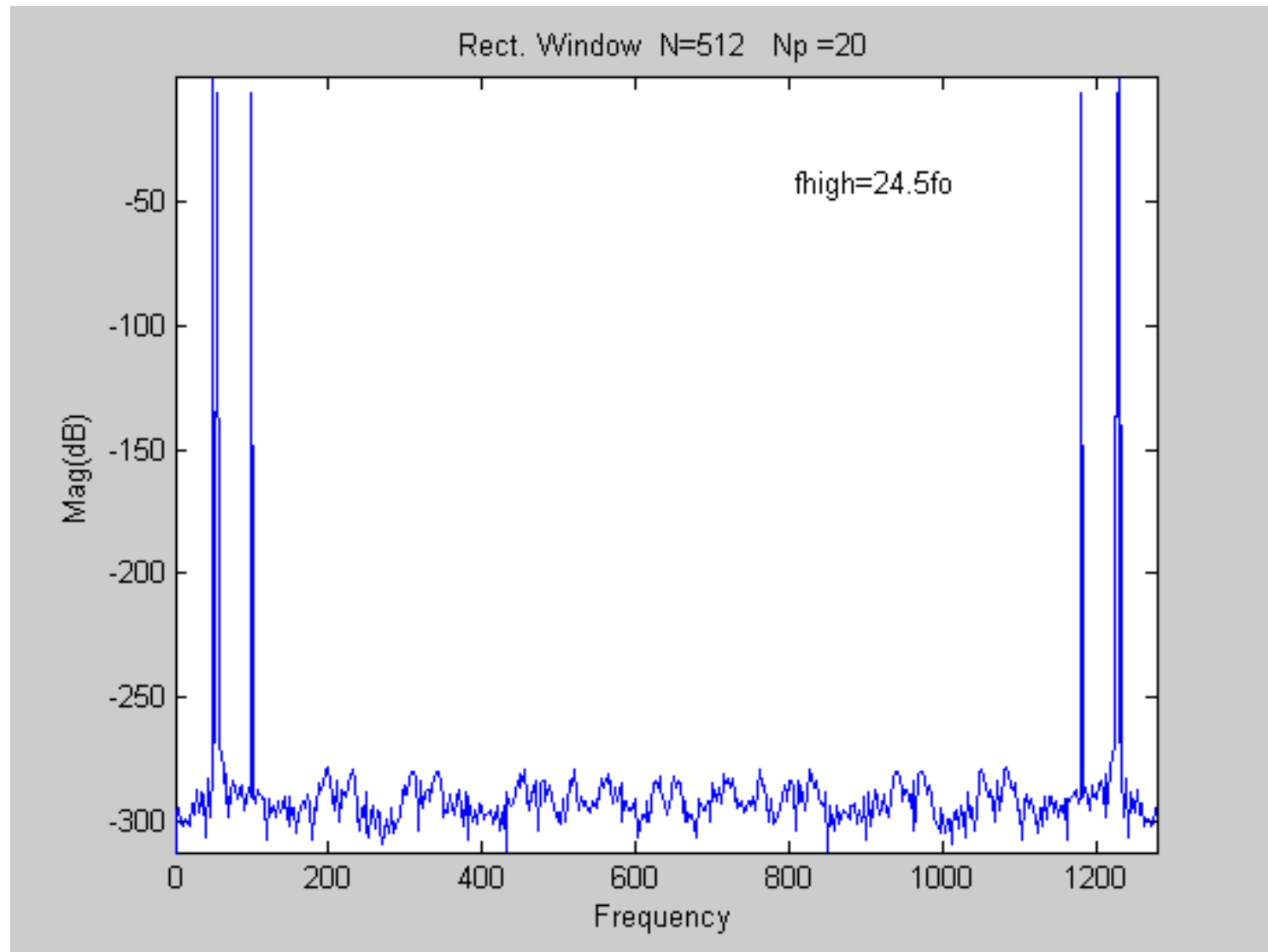


(zoomed in around fundamental)

Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
- Important to avoid aliasing if the DFT is used for spectral characterization

Review Questions

Q1: How many DFT terms are there if the sample window is of length 4096?

A: 4096

Q2: When the magnitude of the DFT coefficients are plotted, the horizontal axis is an index axis (i.e. dimensionless) but often the index terms are labeled as frequency terms. If the sampling frequency is f_s and N samples are taken, what is the frequency of the first DFT term? What is the frequency of the 2nd DFT term?

A: 0 Hz A: f_s/N

Q3: If samples of the time-domain signal are made over exactly 31 periods, which index term corresponds to the fundamental? To the second harmonic?

A: 32nd term A: 63rd term

Q4: What is the difference between the DFT and the FFT?

A: FFT is a computationally efficient method of computing the DFT

Q5: True or False: The DFT terms are real numbers.

A: False We are, however, often interested most in the magnitude of the DFT terms and these are real

Q6: True or False: The magnitude of the DFT terms are symmetric around index number $N/2$. A: Yes

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
 - NP is an integer
 - Band-limited excitation
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Considerations for Spectral Characterization

- Tool Validation
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Are there any strategies to address the problem of requiring precisely an integral number of periods to use the FFT?

Windowing is sometimes used

Windowing is sometimes misused

Windowing

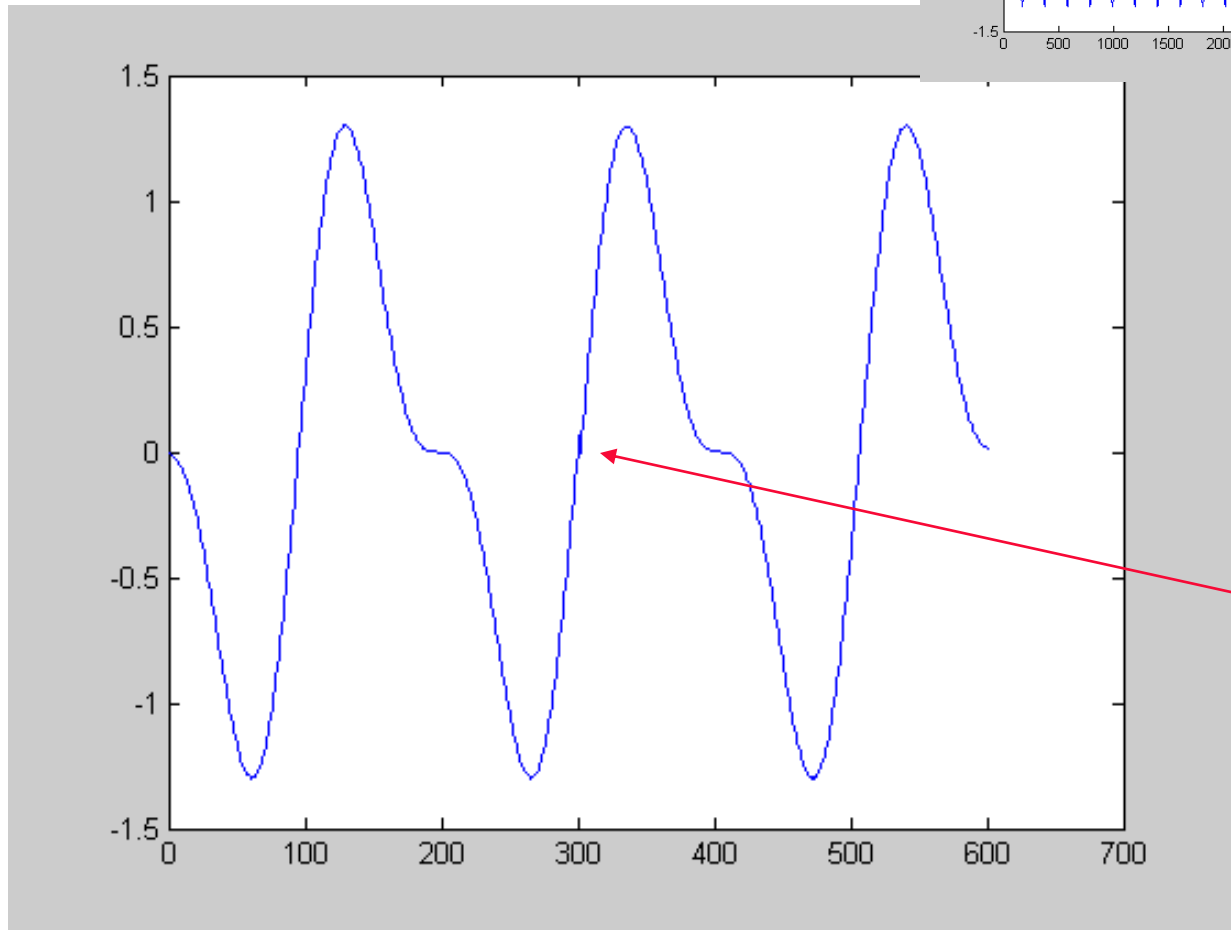
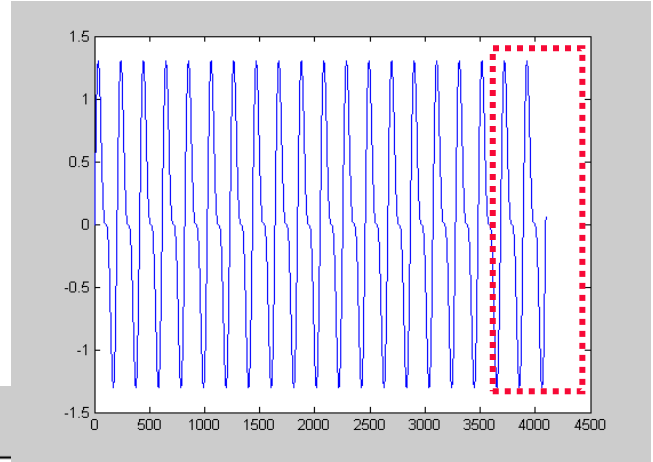
Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window

Well-studied window functions:

- Rectangular (also with appended zeros)
- Triangular
- Hamming
- Hanning
- Blackman

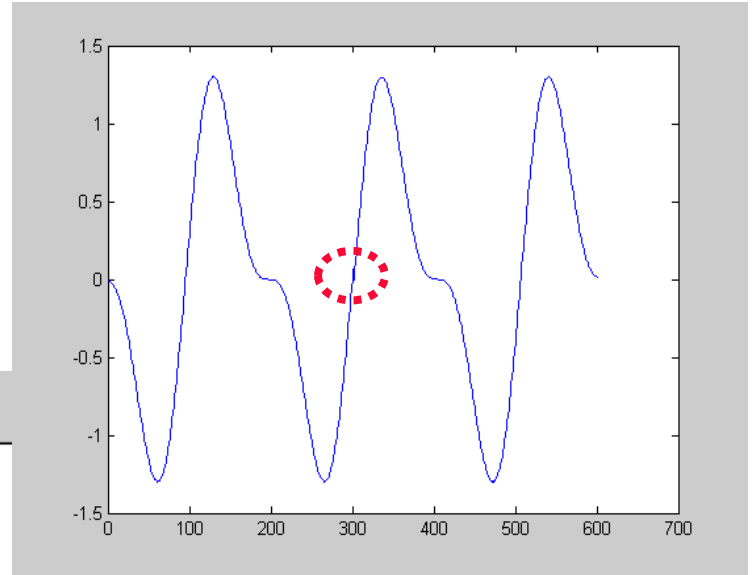
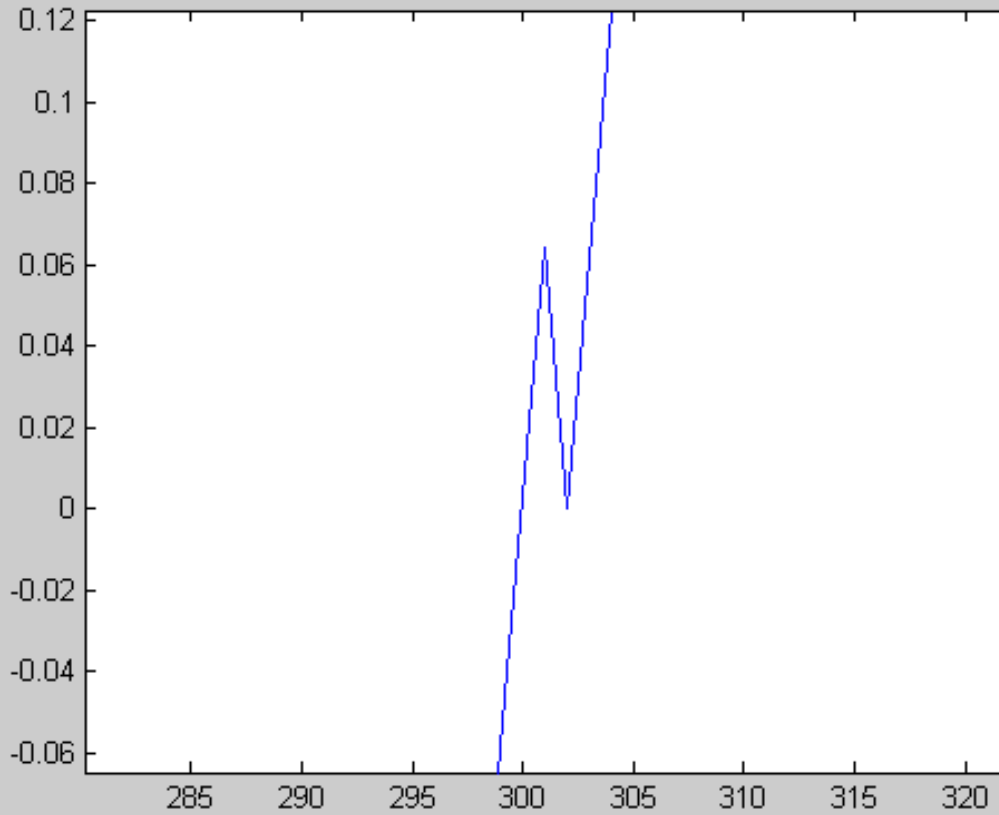
Recall

Input Waveform



Recall

Input Waveform



Rectangular Window

Sometimes termed a boxcar window

Uniform weight

Can append zeros

Without appending zeros equivalent to no window

Rectangular Window

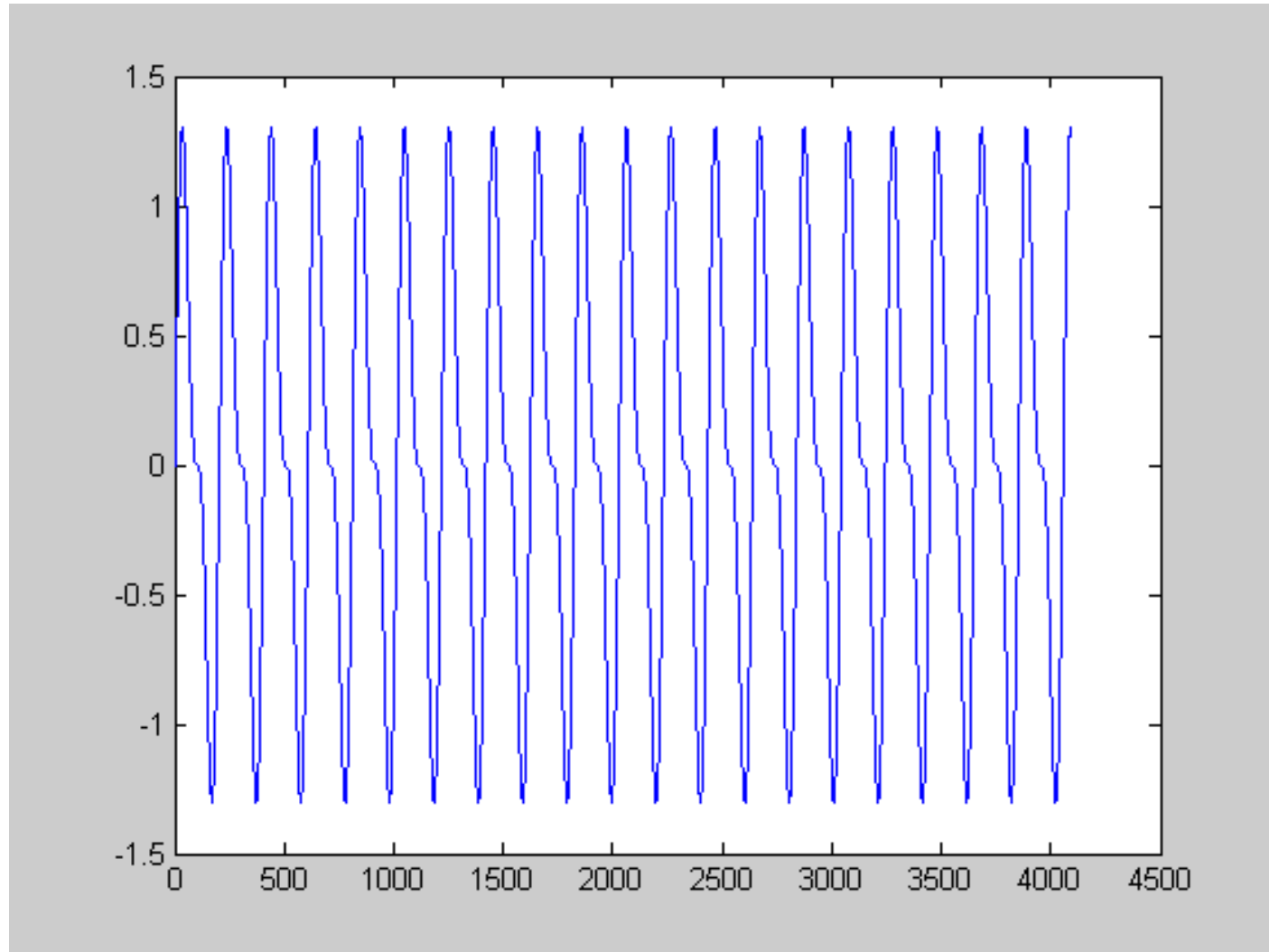
Assume $f_{\text{SIG}}=50\text{Hz}$

$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

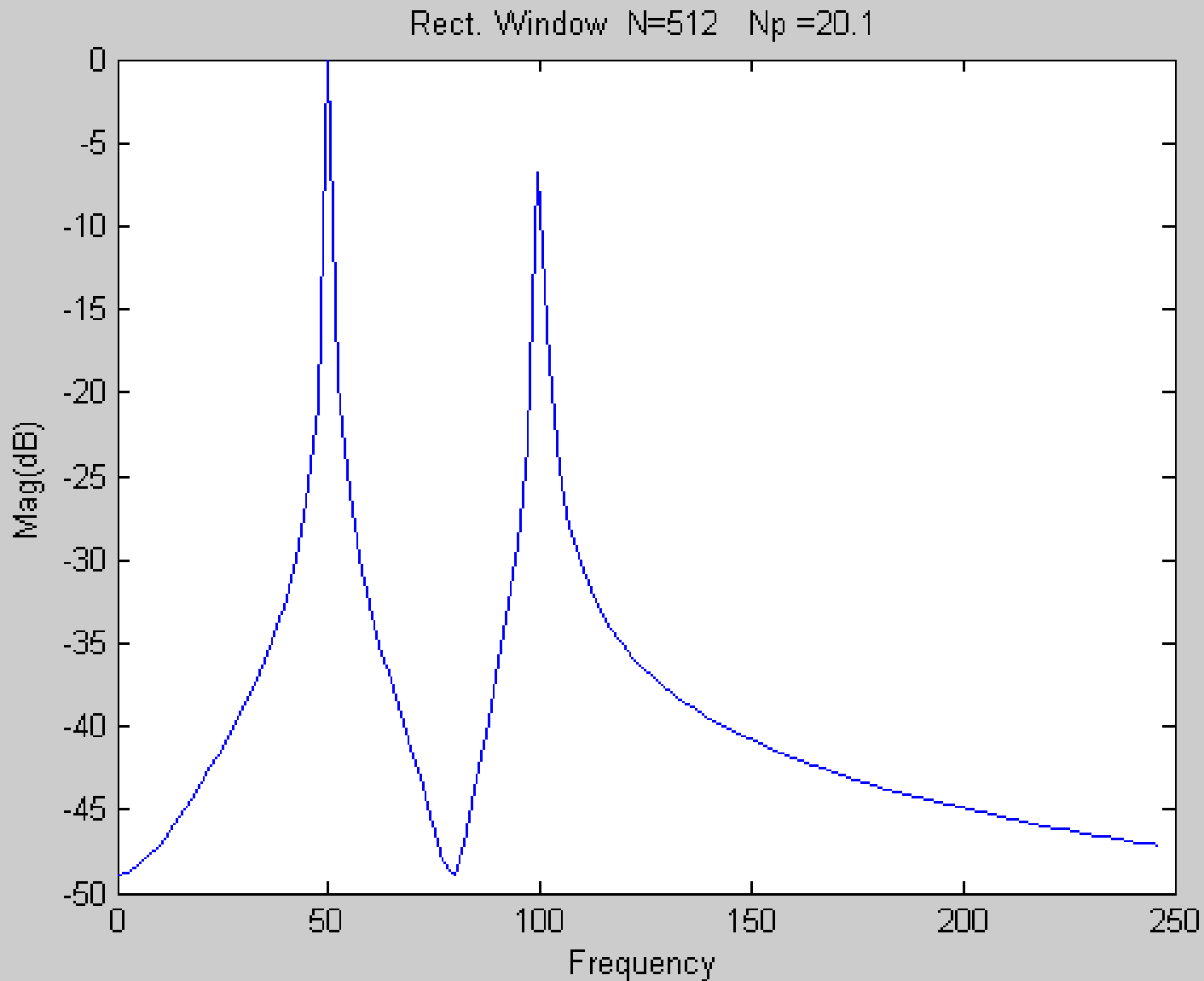
$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_p=20.1$ $N=512$

Rectangular Window

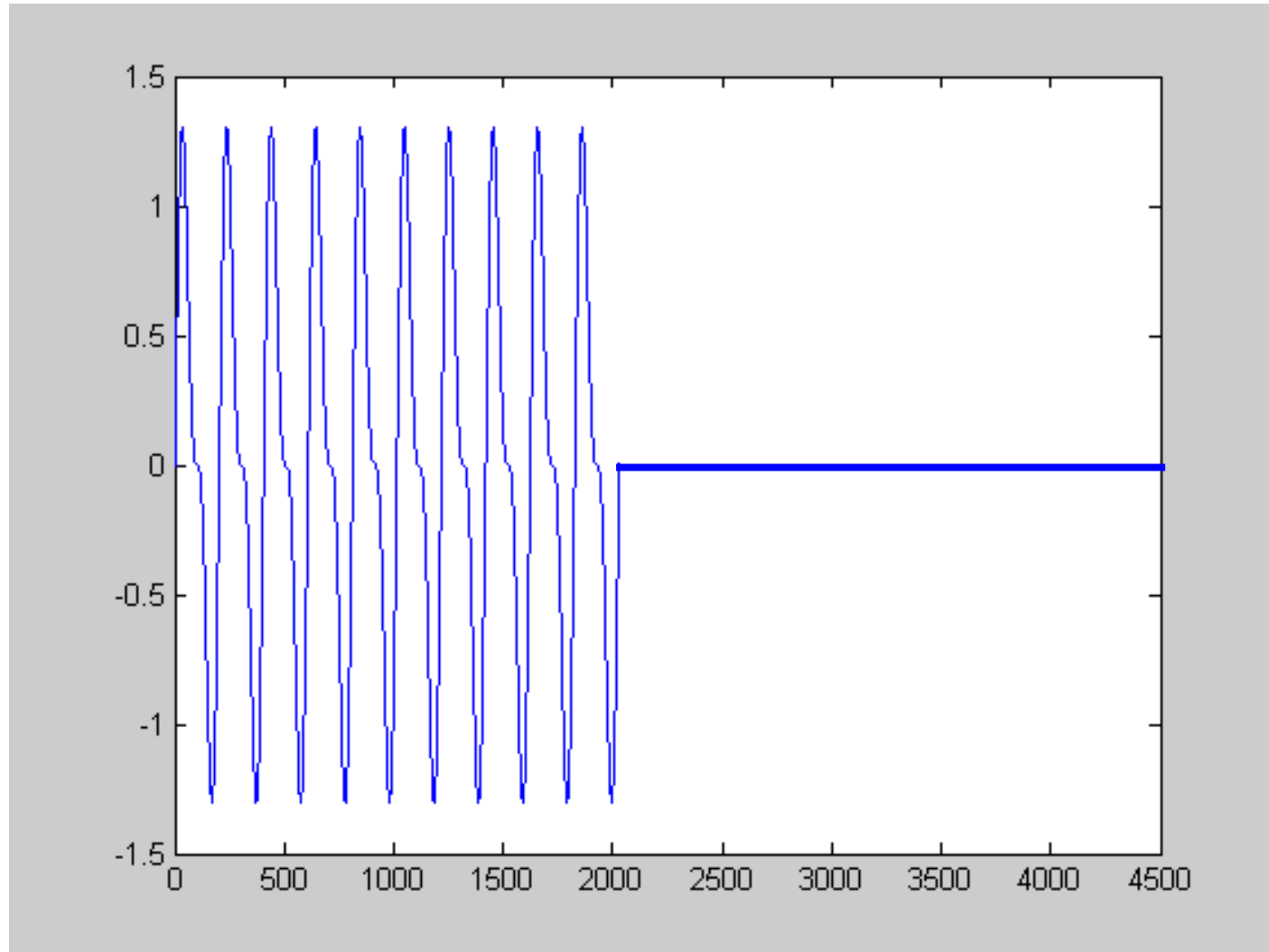


Spectral Response with Non-coherent sampling



(zoomed in around fundamental)

Rectangular Window (with appended zeros)



Rectangular Window

Columns 1 through 7

-48.8444 -48.7188 -48.3569 -47.7963 -47.0835 -46.2613 -45.3620

Columns 8 through 14

-44.4065 -43.4052 -42.3602 -41.2670 -40.1146 -38.8851 -37.5520

Columns 15 through 21

-36.0756 -34.3940 -32.4043 -29.9158 -26.5087 -20.9064 -0.1352

Columns 22 through 28

-19.3242 -25.9731 -29.8688 -32.7423 -35.1205 -37.2500 -39.2831

Columns 29 through 35

-41.3375 -43.5152 -45.8626 -48.0945 -48.8606 -46.9417 -43.7344

Rectangular Window

Columns 1 through 7

-48.8444 -48.7188 -48.3569 -47.7963 -47.0835 -46.2613 -45.3620

Columns 8 through 14

-44.4065 -43.4052 -42.3602 -41.2670 -40.1146 -38.8851 -37.5520

Columns 15 through 21

-36.0756 -34.3940 -32.4043 29.9158 -26.5087 -20.9064 -0.1352

Columns 22 through 28

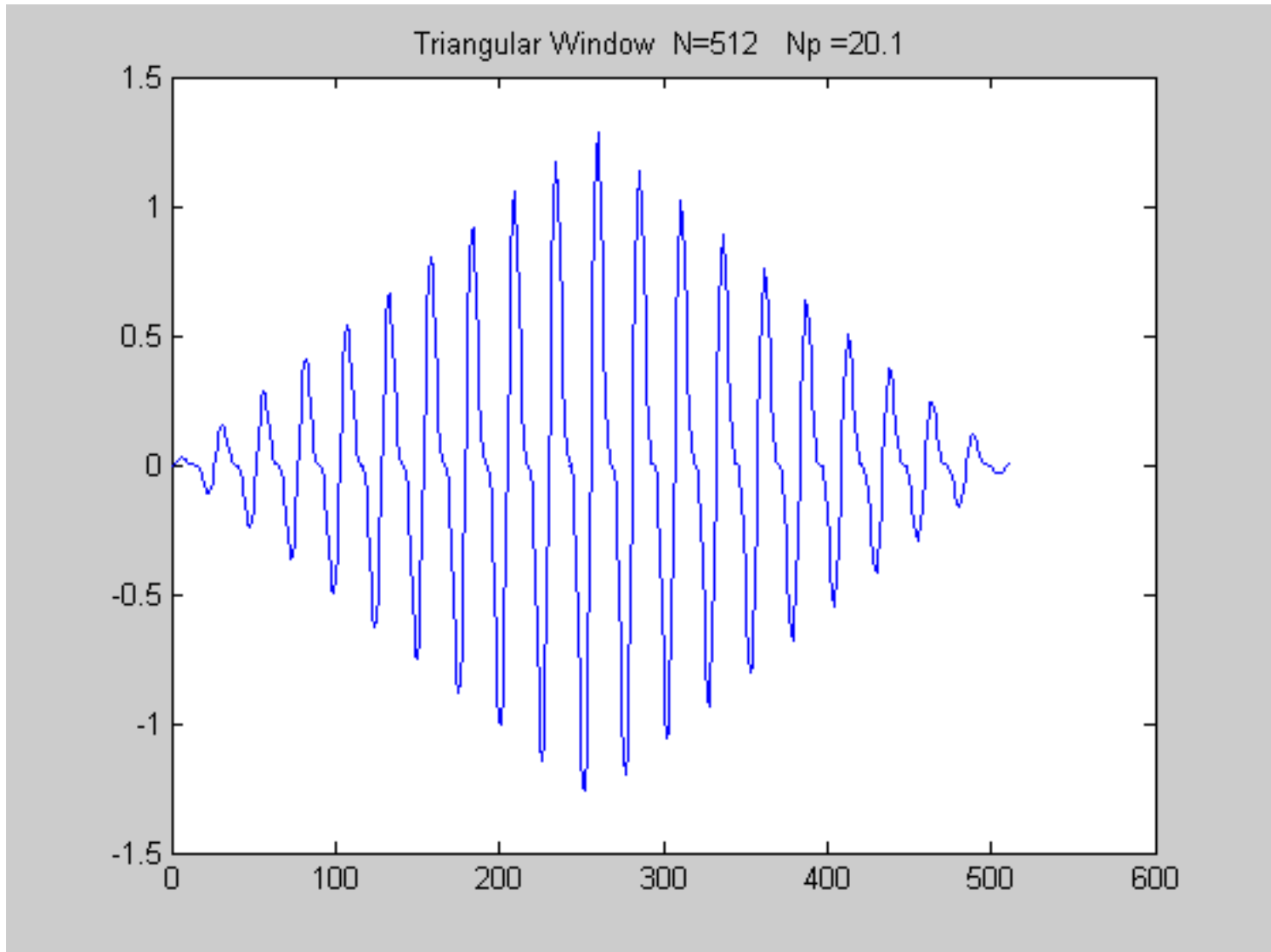
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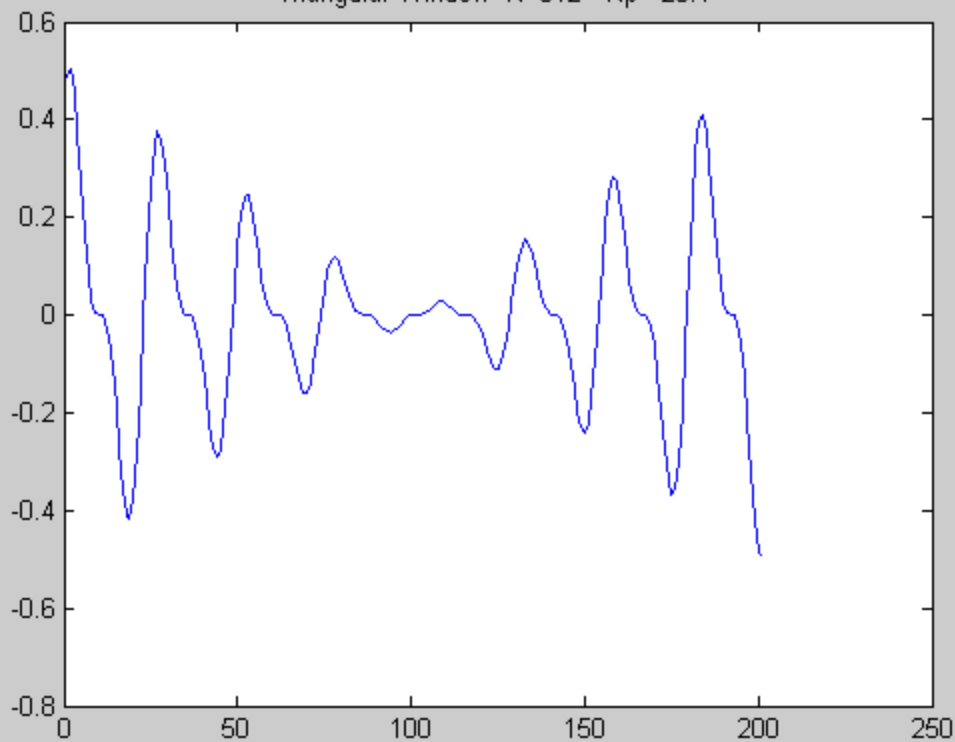
Energy spread over several frequency components

Triangular Window

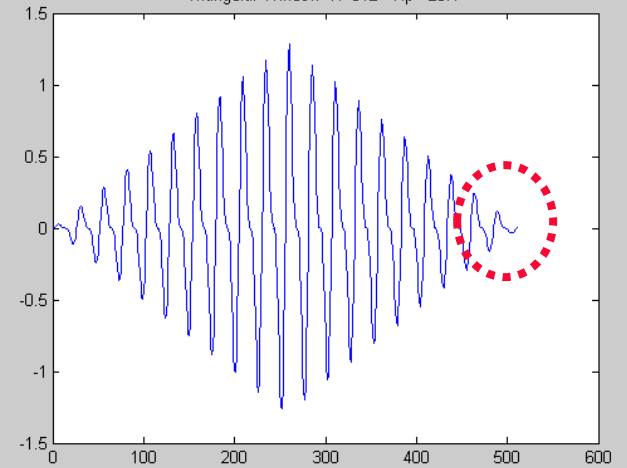


Triangular Window

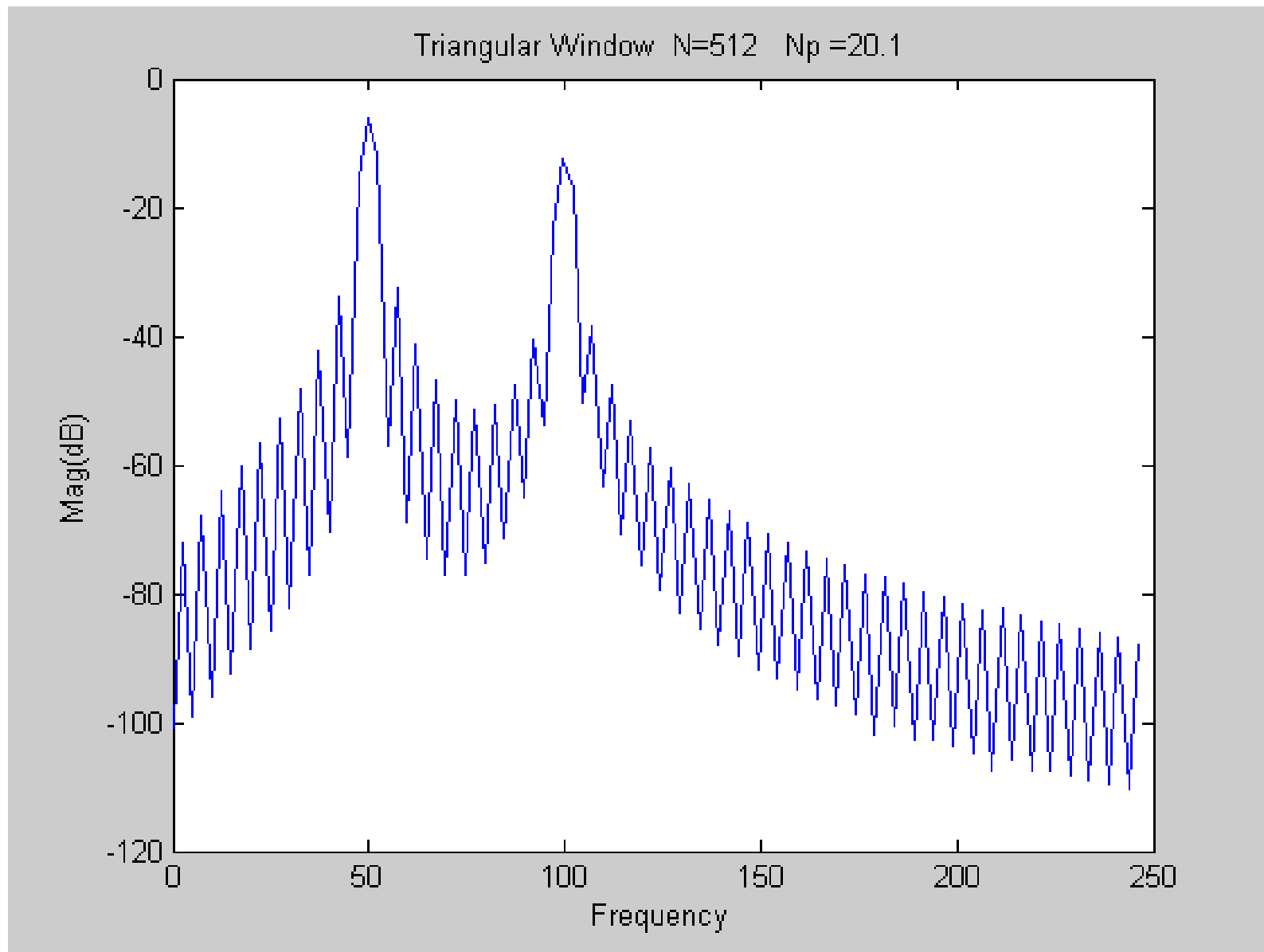
Triangular Window N=512 Np =20.1



Triangular Window N=512 Np =20.1

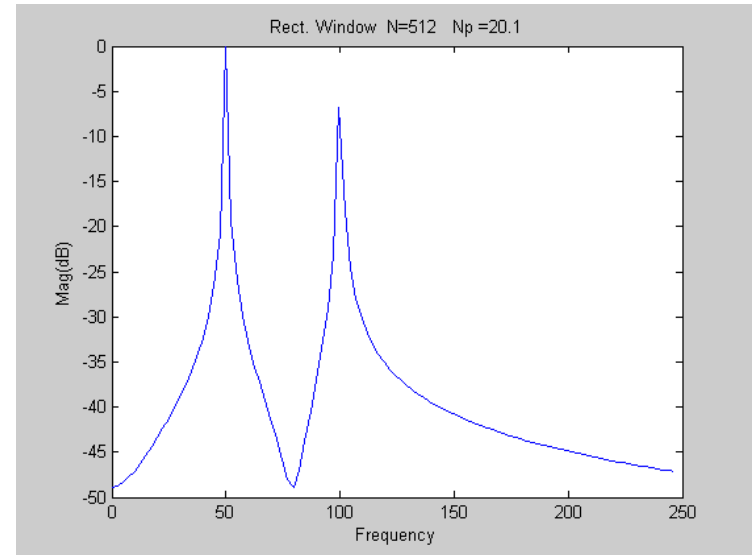
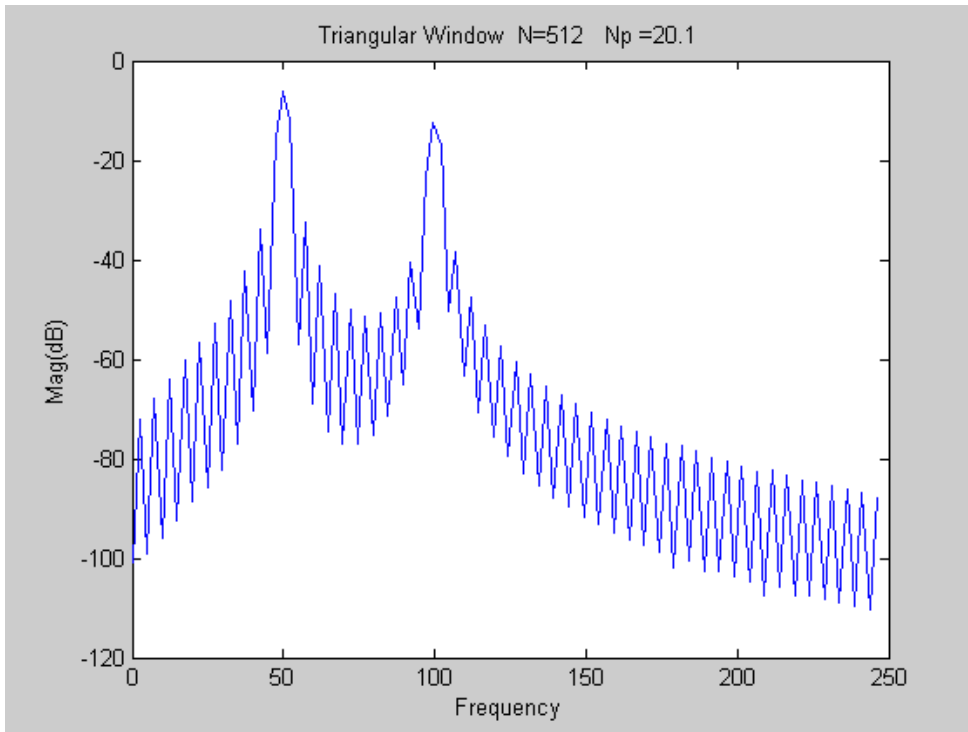


Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Triangular Window



Triangular Window

Columns 1 through 7

-100.8530 -72.0528 -99.1401 -68.0110 -95.8741 -63.9944 -92.5170

Columns 8 through 14

-60.3216 -88.7000 -56.7717 -85.8679 -52.8256 -82.1689 -48.3134

Columns 15 through 21

-77.0594 -42.4247 -70.3128 -33.7318 -58.8762 -15.7333 -6.0918

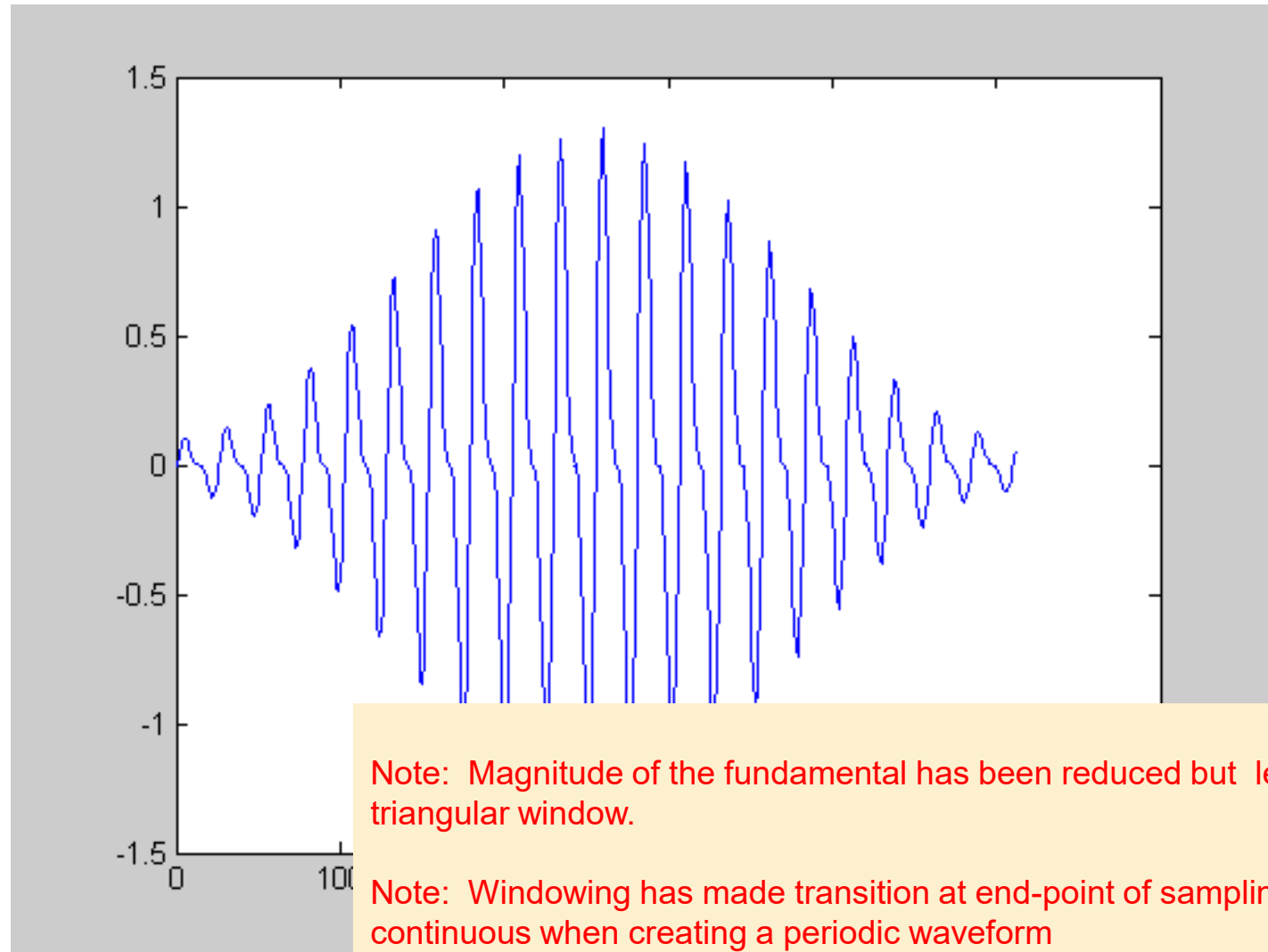
Colt

-12. Note: Magnitude of the fundamental has been reduced but the skirting effects have also been reduced.

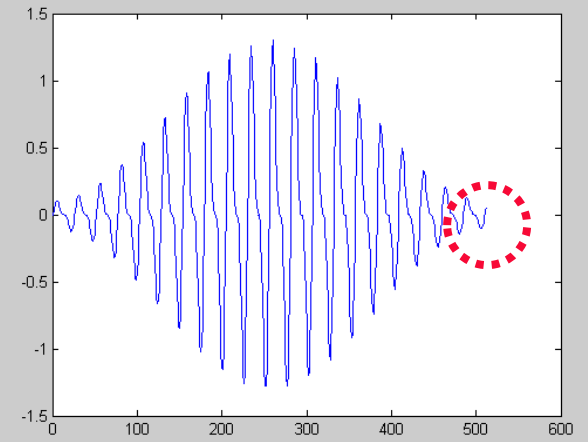
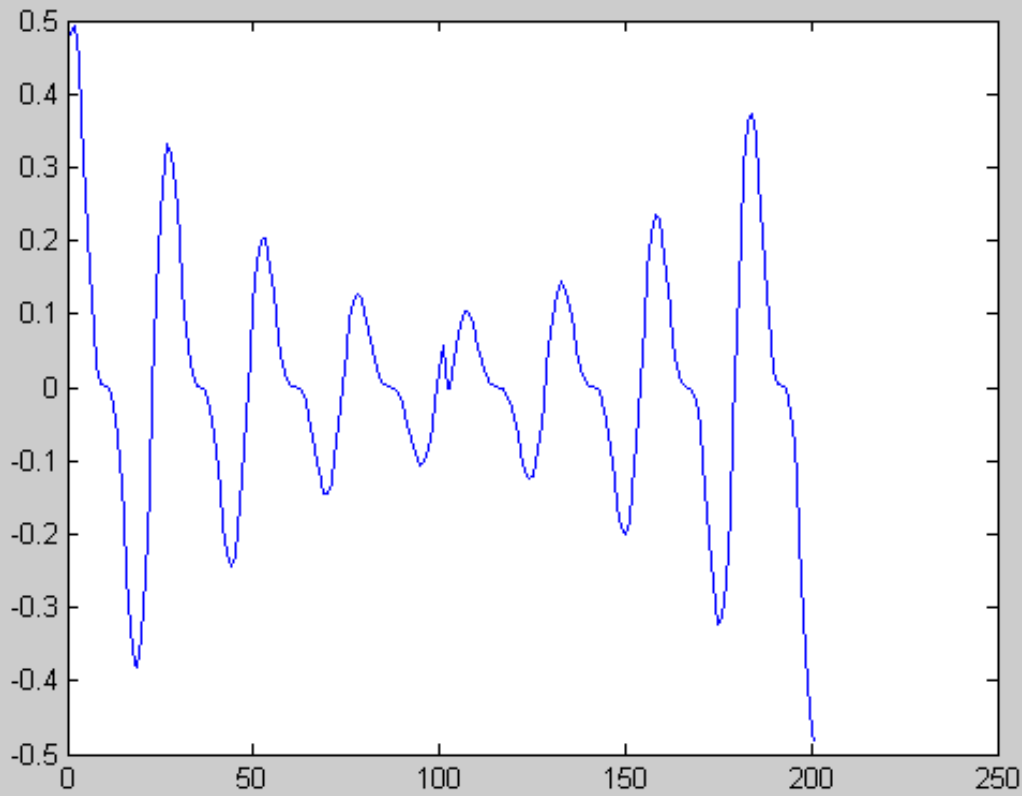
Colt

-77. Note: Windowing has reduced energy in the signal but also made transition at end-point of sampling window continuous when creating a periodic waveform

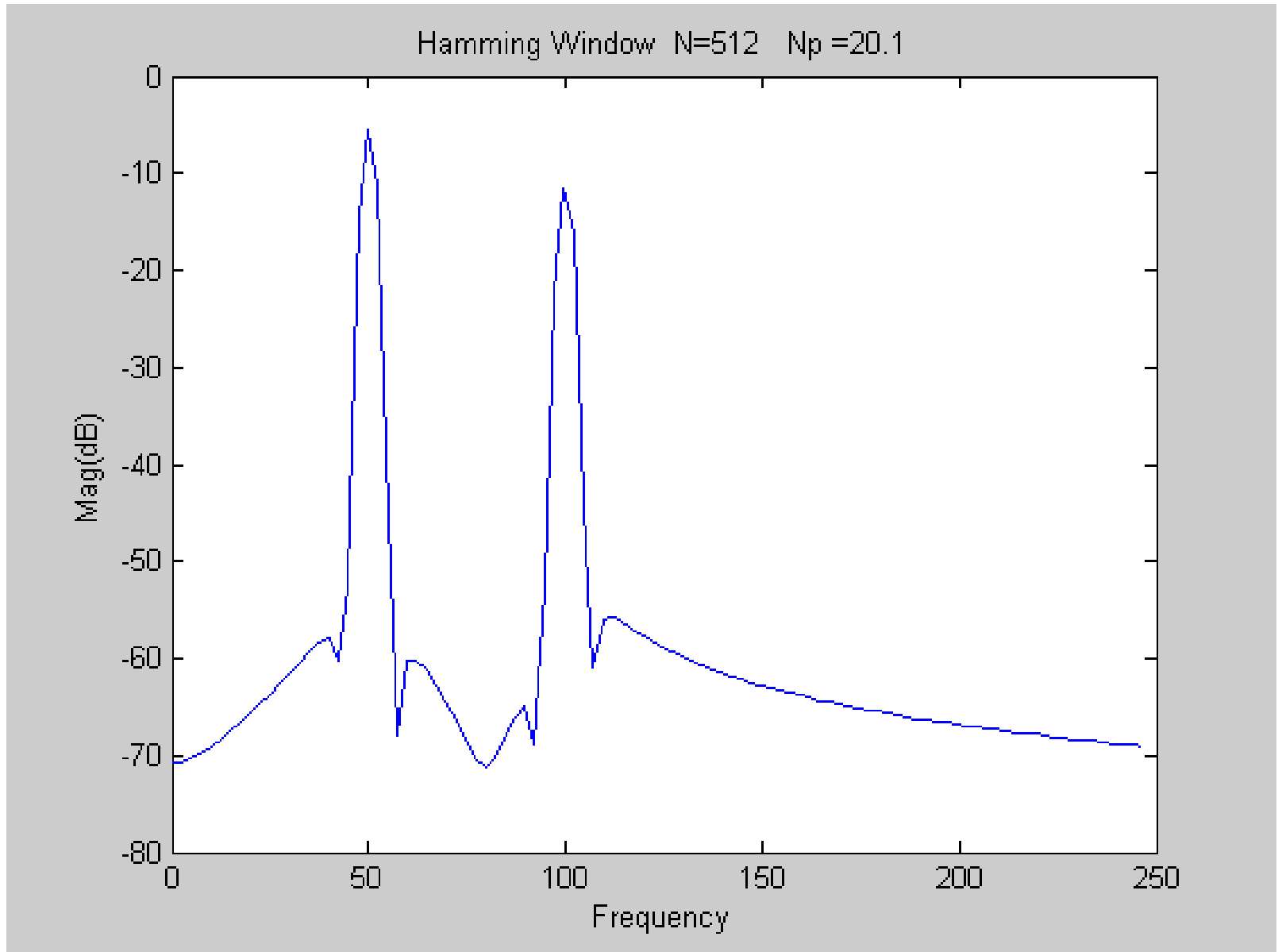
Hamming Window



Hamming Window

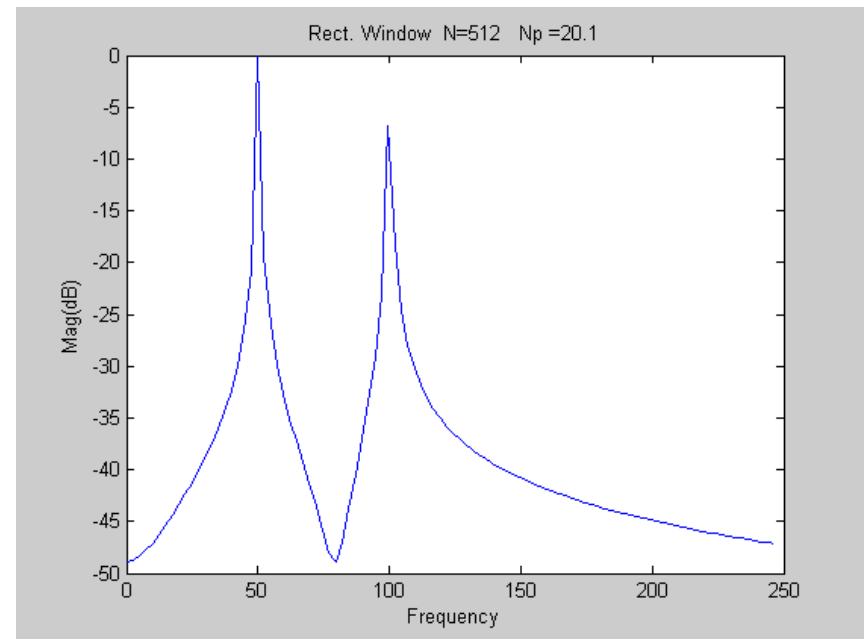
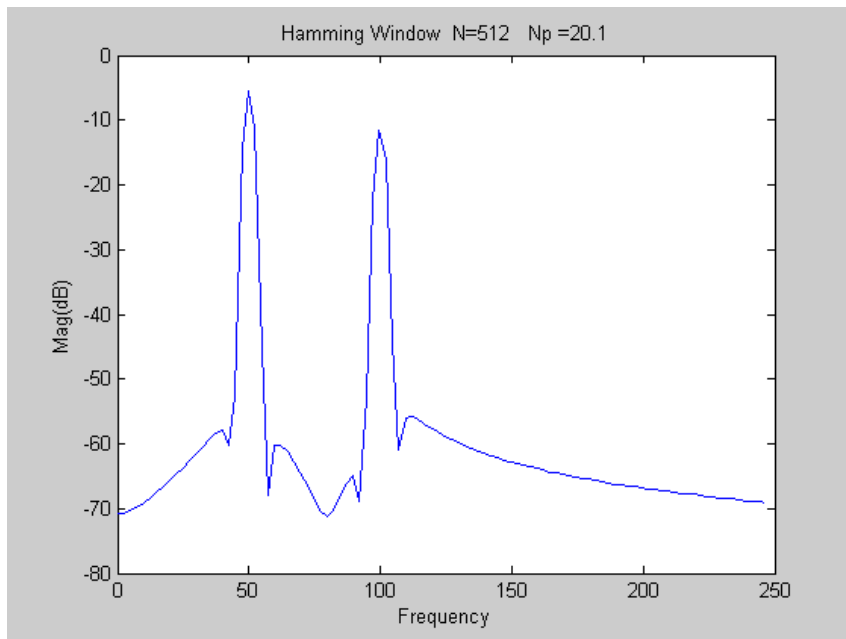


Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Comparison with Rectangular Window



Note: Vertical axis are different

Hamming Window

Columns 1 through 7

-70.8278 -70.6955 -70.3703 -69.8555 -69.1502 -68.3632 -67.5133

Columns 8 through 14

-66.5945 -65.6321 -64.6276 -63.6635 -62.6204 -61.5590 -60.4199

Columns 15 through 21

-59.3204 -58.3582 -57.8735 -60.2994 -52.6273 -14.4702 -5.4343

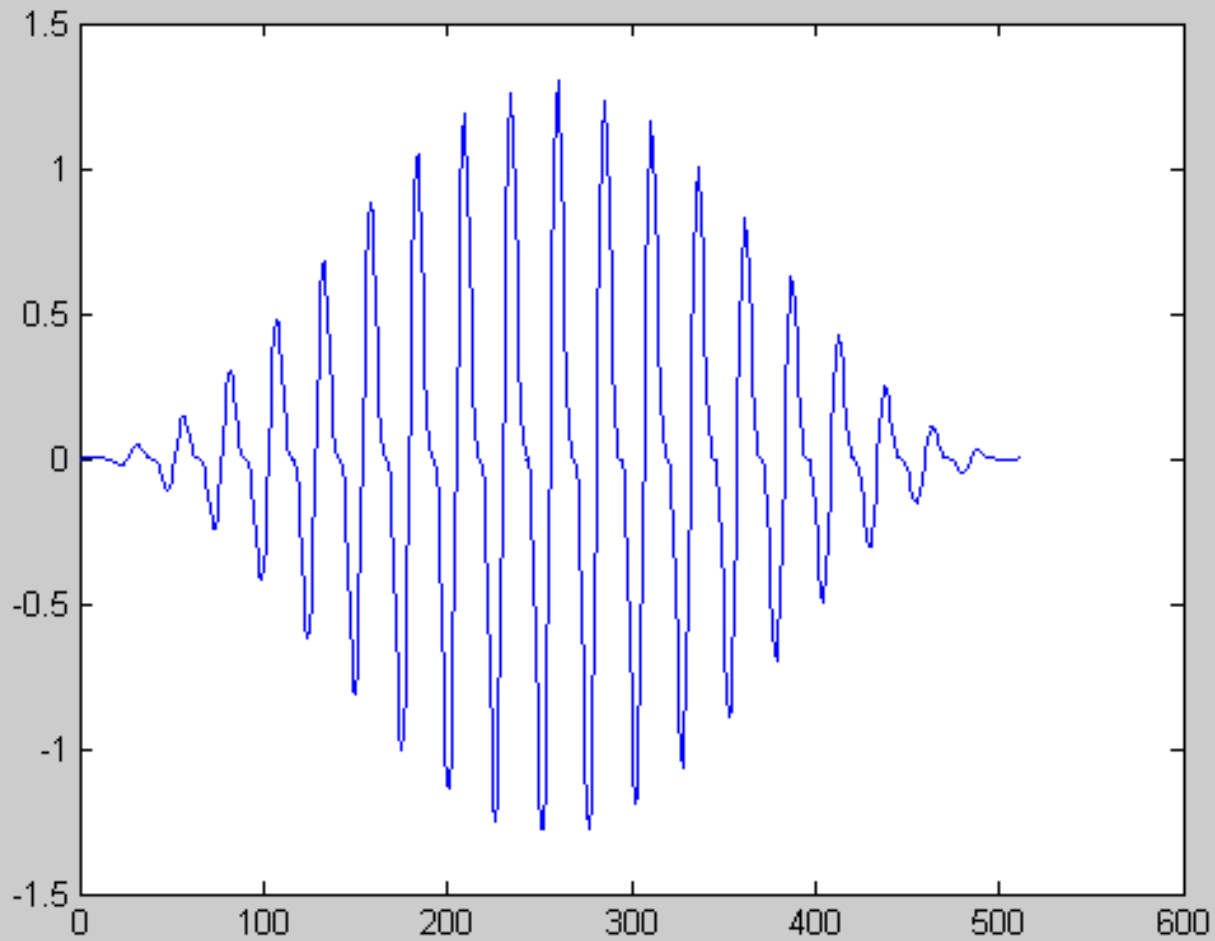
Columns 22 through 28

-11.2659 -45.2190 -67.9926 -60.1662 -60.1710 -61.2796 -62.7277

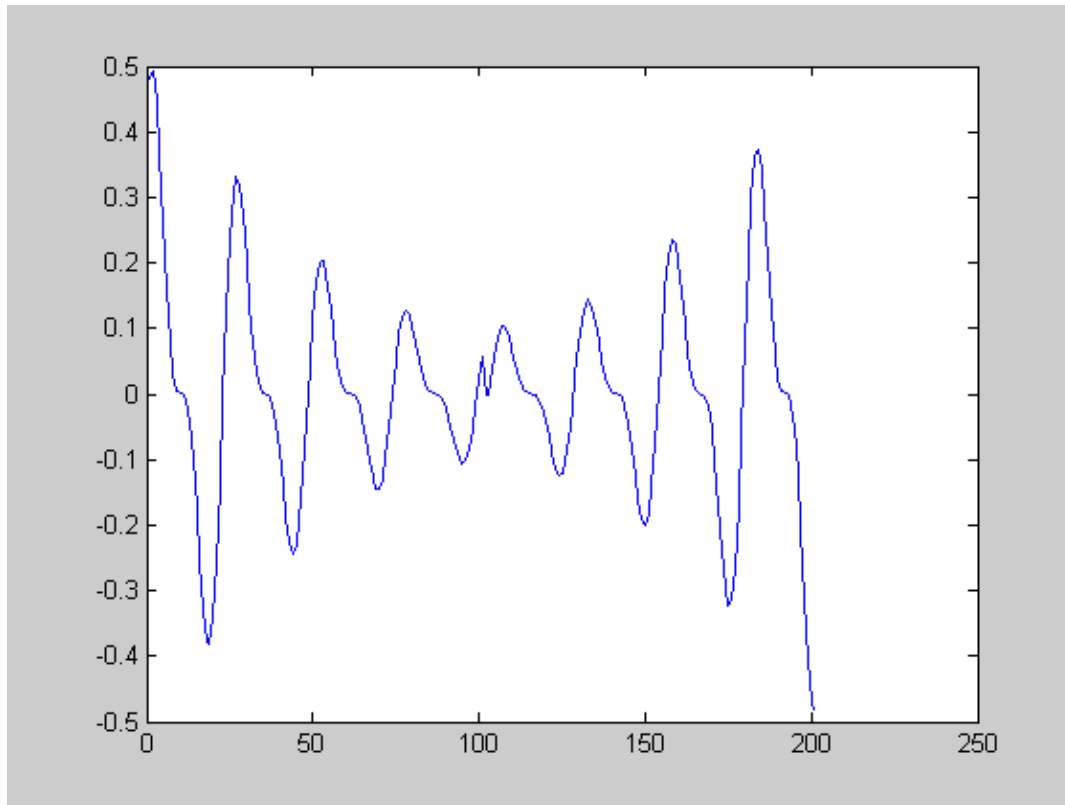
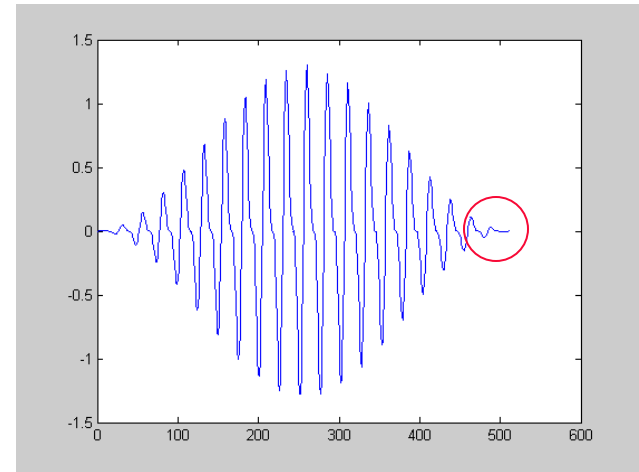
Columns 29 through 35

-64.3642 -66.2048 -68.2460 -70.1835 -71.1529 -70.2800 -68.1145

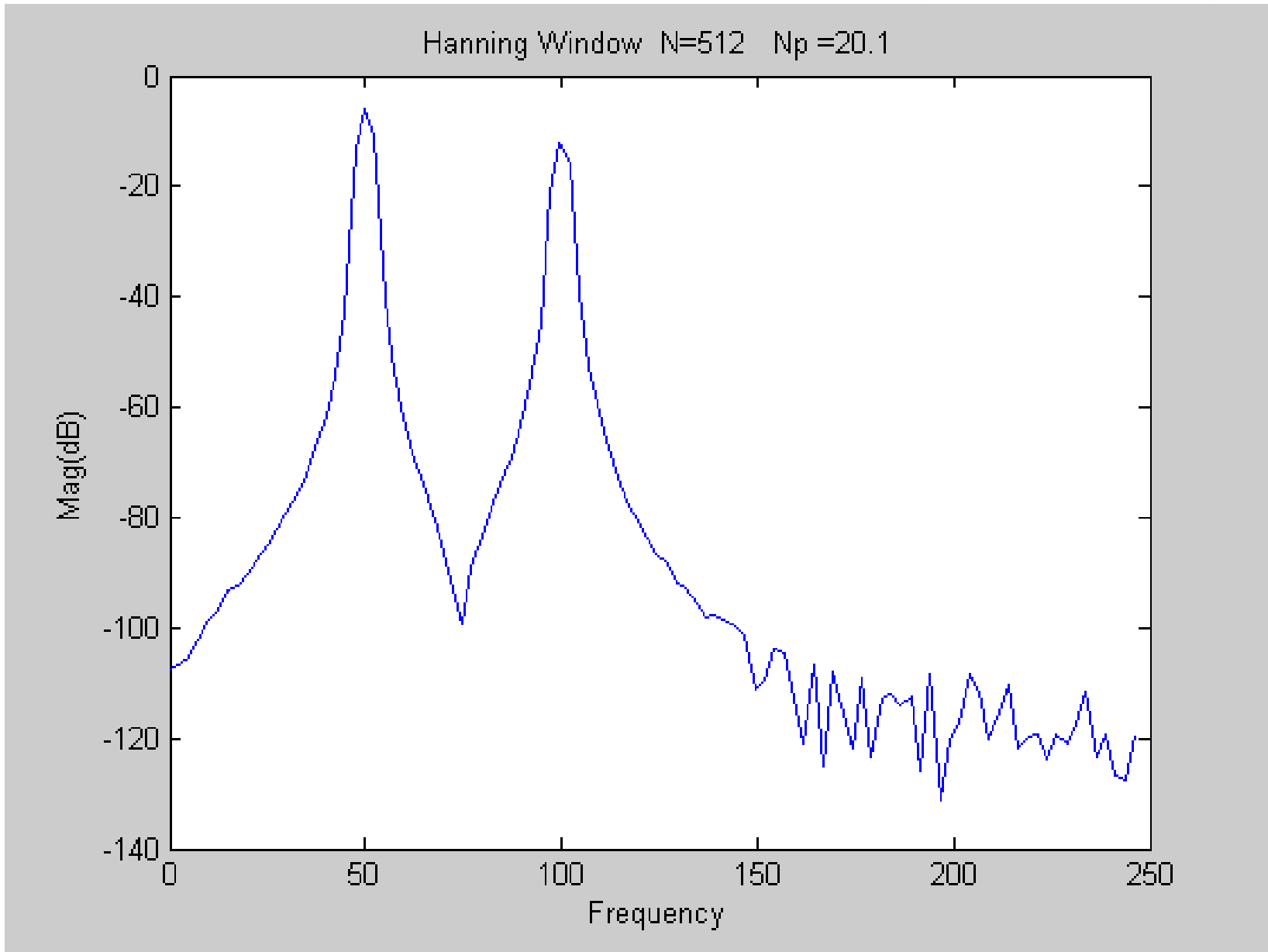
Hanning Window



Hanning Window

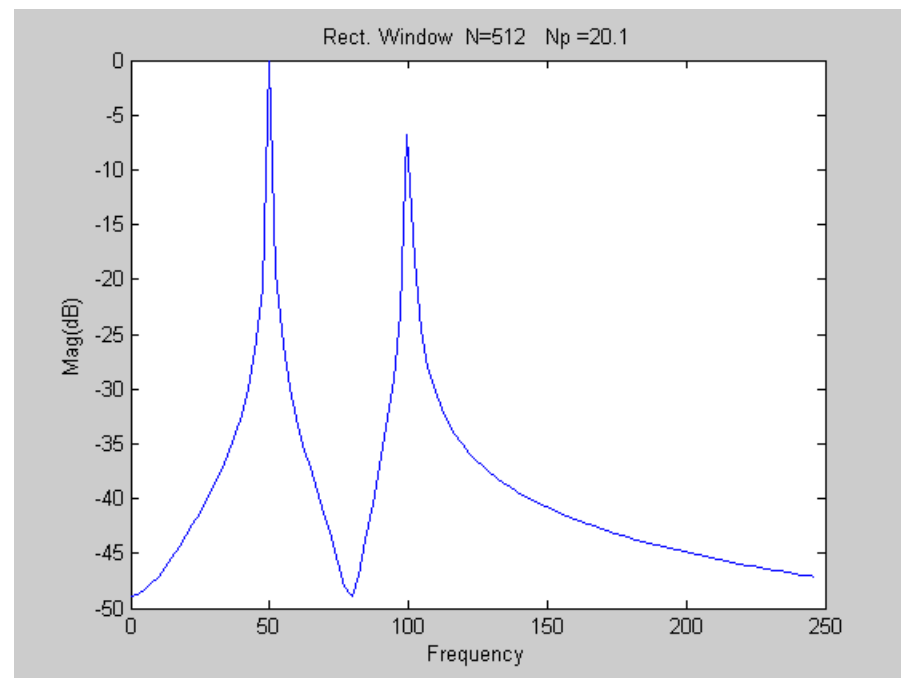
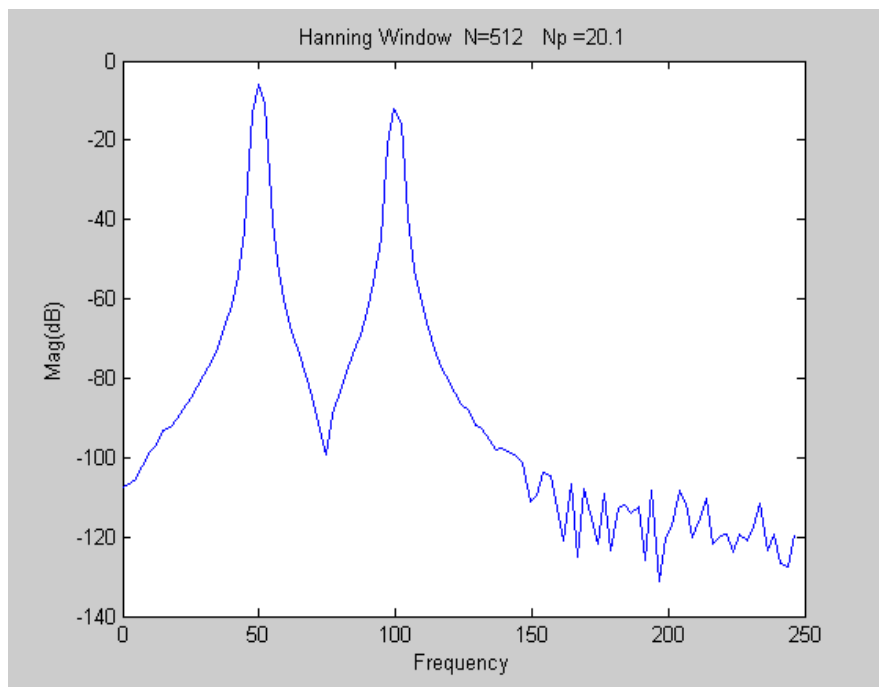


Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Comparison with Rectangular Window



Note: Vertical axis are different

Hanning Window

Columns 1 through 7

-107.3123 -106.7939 -105.3421 -101.9488 -98.3043 -96.6522 -93.0343

Columns 8 through 14

-92.4519 -90.4372 -87.7977 -84.9554 -81.8956 -79.3520 -75.8944

Columns 15 through 21

-72.0479 -67.4602 -61.7543 -54.2042 -42.9597 -13.4511 -6.0601

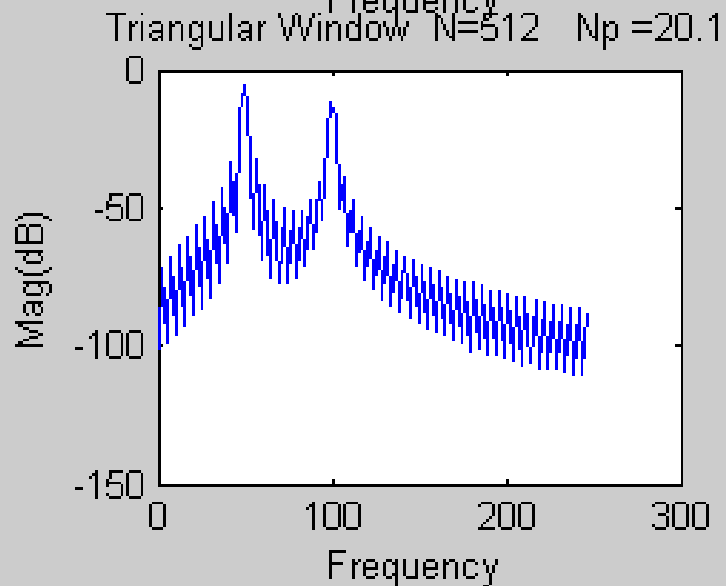
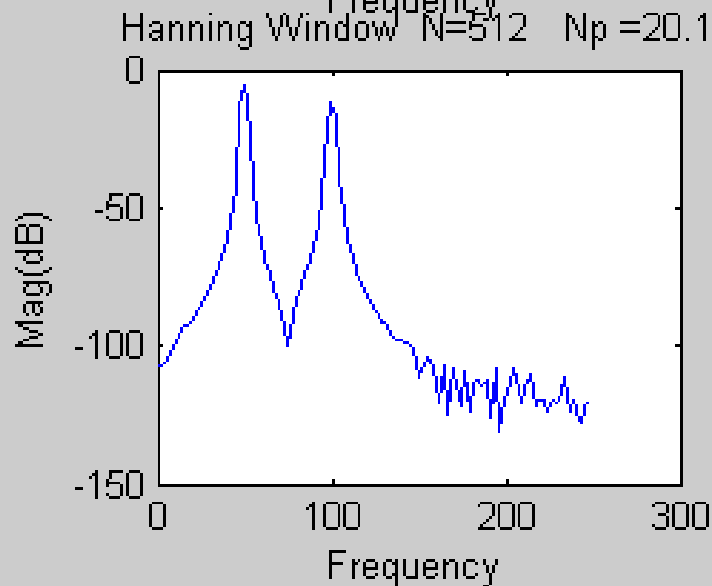
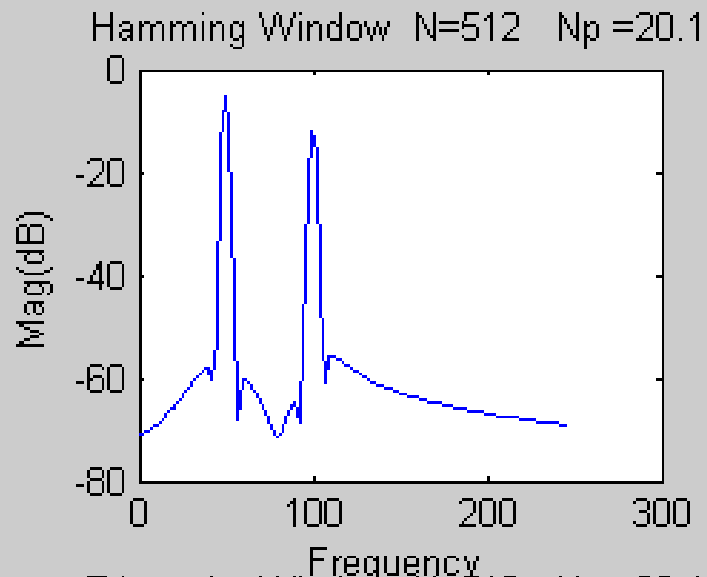
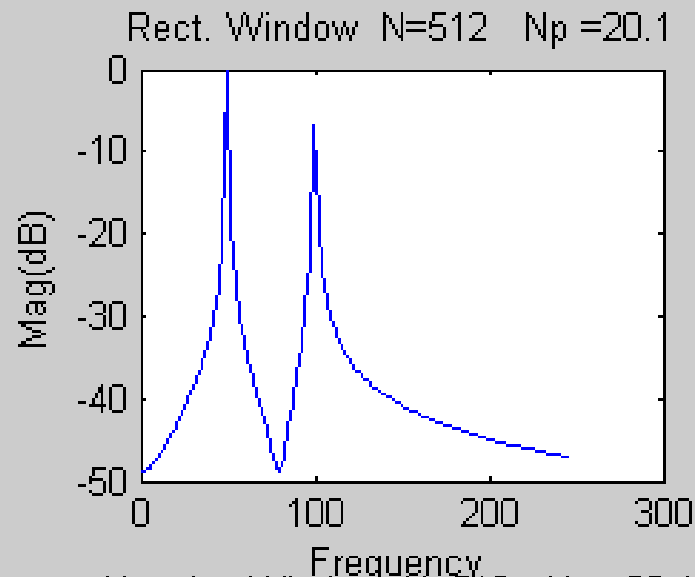
Columns 22 through 28

-10.8267 -40.4480 -53.3906 -61.8561 -68.3601 -73.9966 -79.0757

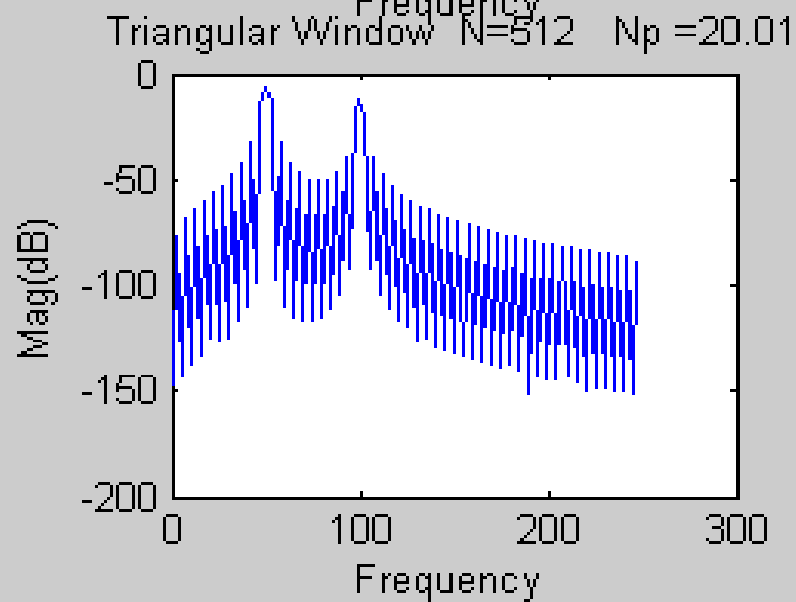
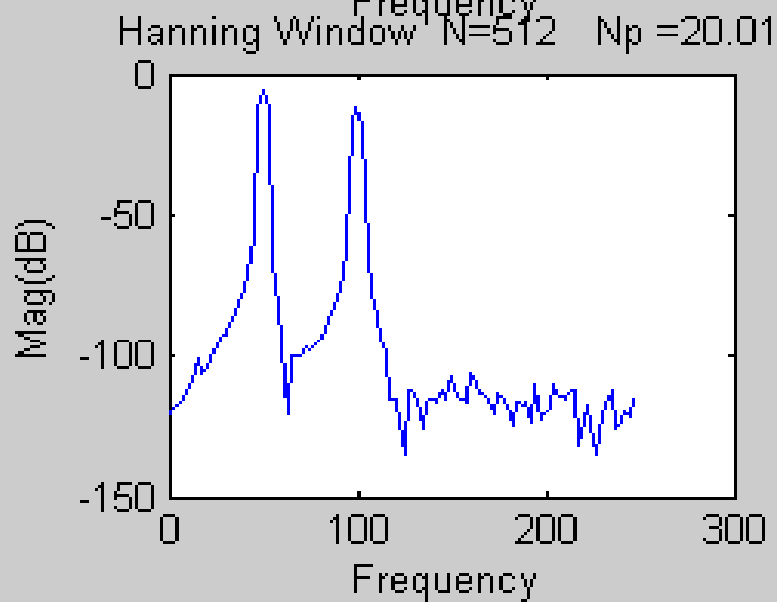
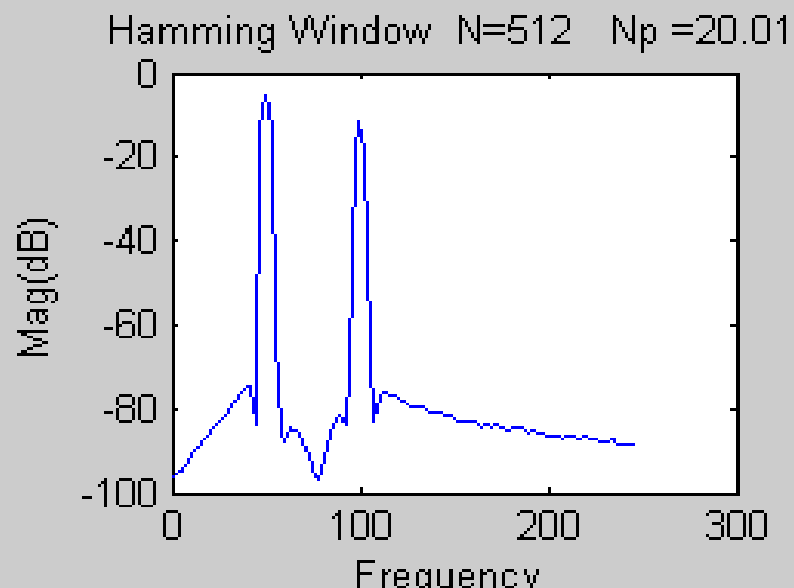
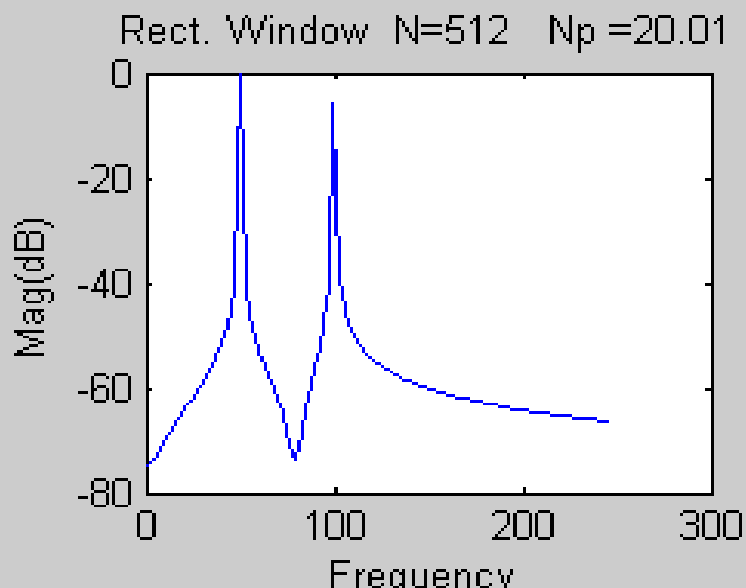
Columns 29 through 35

-84.4318 -92.7280 -99.4046 -89.0799 -83.4211 -78.5955 -73.9788

Comparison of 4 windows



Comparison of 4 windows

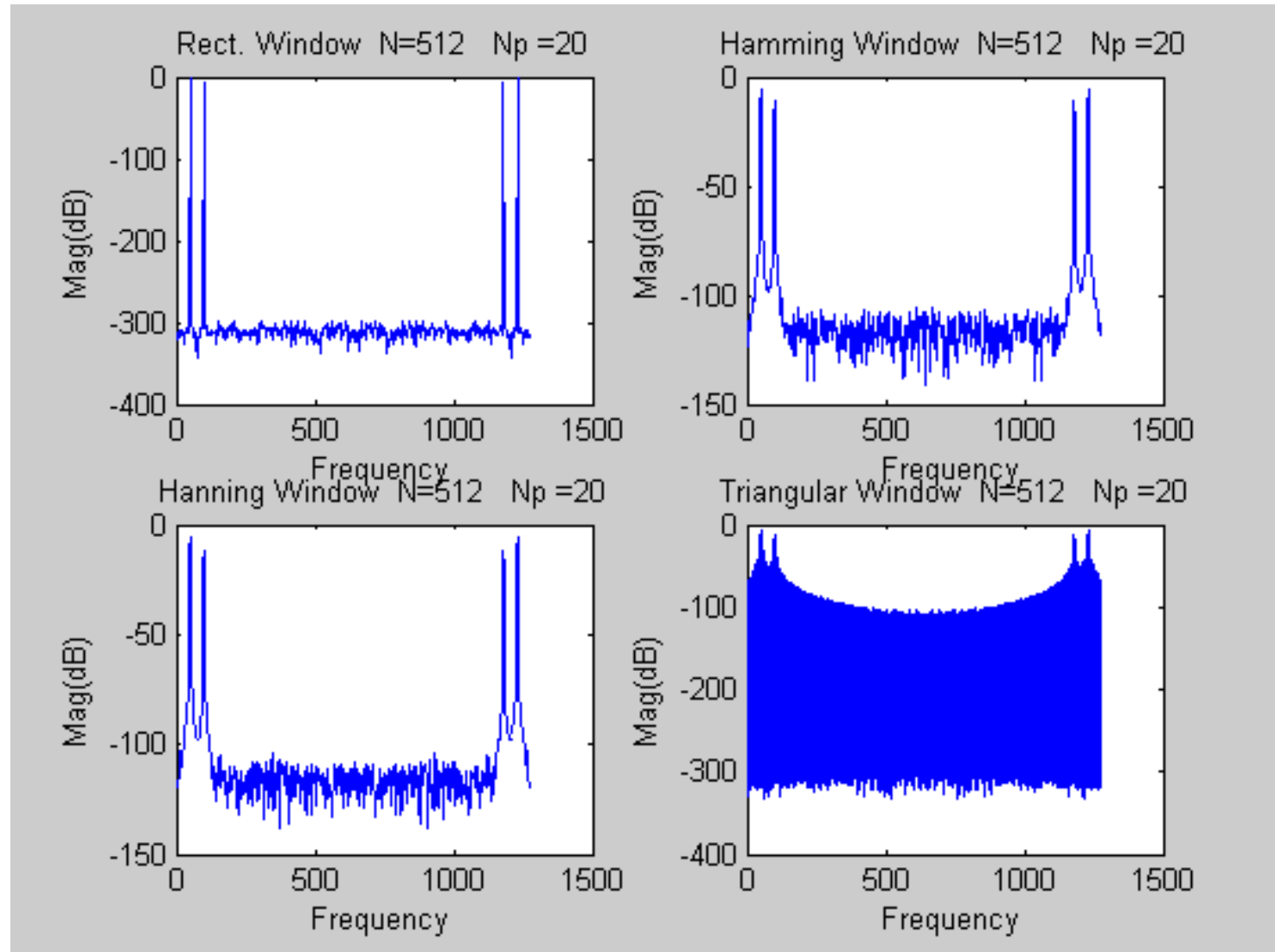


But windows can make things worse too!

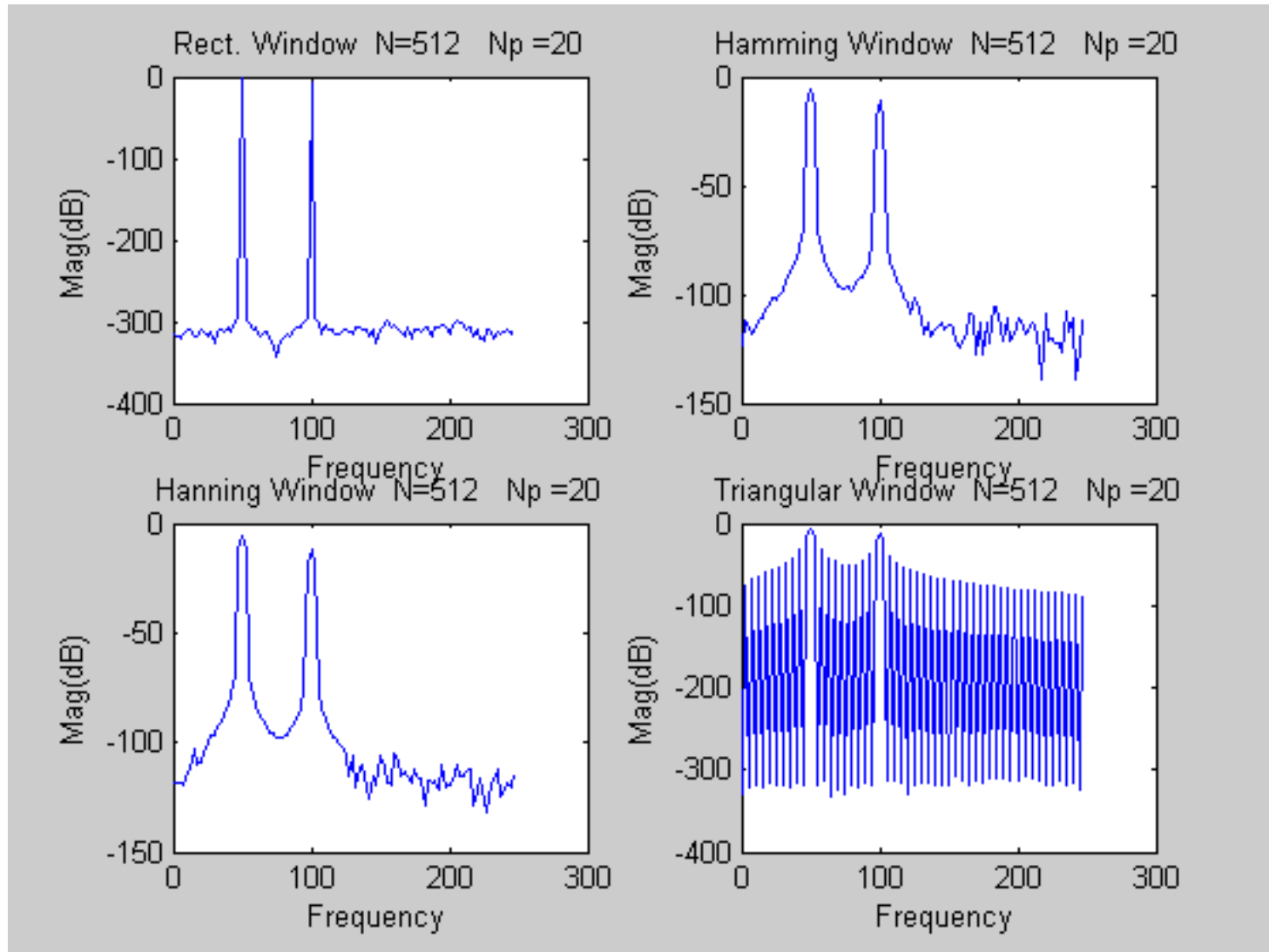
Consider situation where we really do have coherent sampling and a window is applied

fsig1=50Hz
fsig2=100Hz
N=512
Np=20

Comparison of 4 windows when sampling hypothesis are satisfied



Comparison of 4 windows



But windows can make things worse too!

Consider situation where we really do have coherent sampling and a window is applied

f_{sig1}=50Hz
f_{sig2}=100Hz
N=512
N_p=20

And we do not really know how much worse thing can be!

Be careful about interpreting results obtained by using windowing to mitigate the non-coherent sampling problem !

Remember the hypothesis of the theorem relating the DFT, which is easy to calculate, to the Fourier Series coefficients!

Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met

Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis

Not easy to satisfy this requirement in the laboratory

Windowing can help but can hurt as well

Out of band energy can be reflected back into bands of interest

Characterization of CAD tool environment is essential

Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance

Spectral Characterization of Data Converters

- Distortion Analysis



Time Quantization Effects

- of DACs
 - of ADCs
- Amplitude Quantization Effects
 - of DACs
 - of ADCs

Quantization Effects on Spectral Performance and Noise Floor in DFT

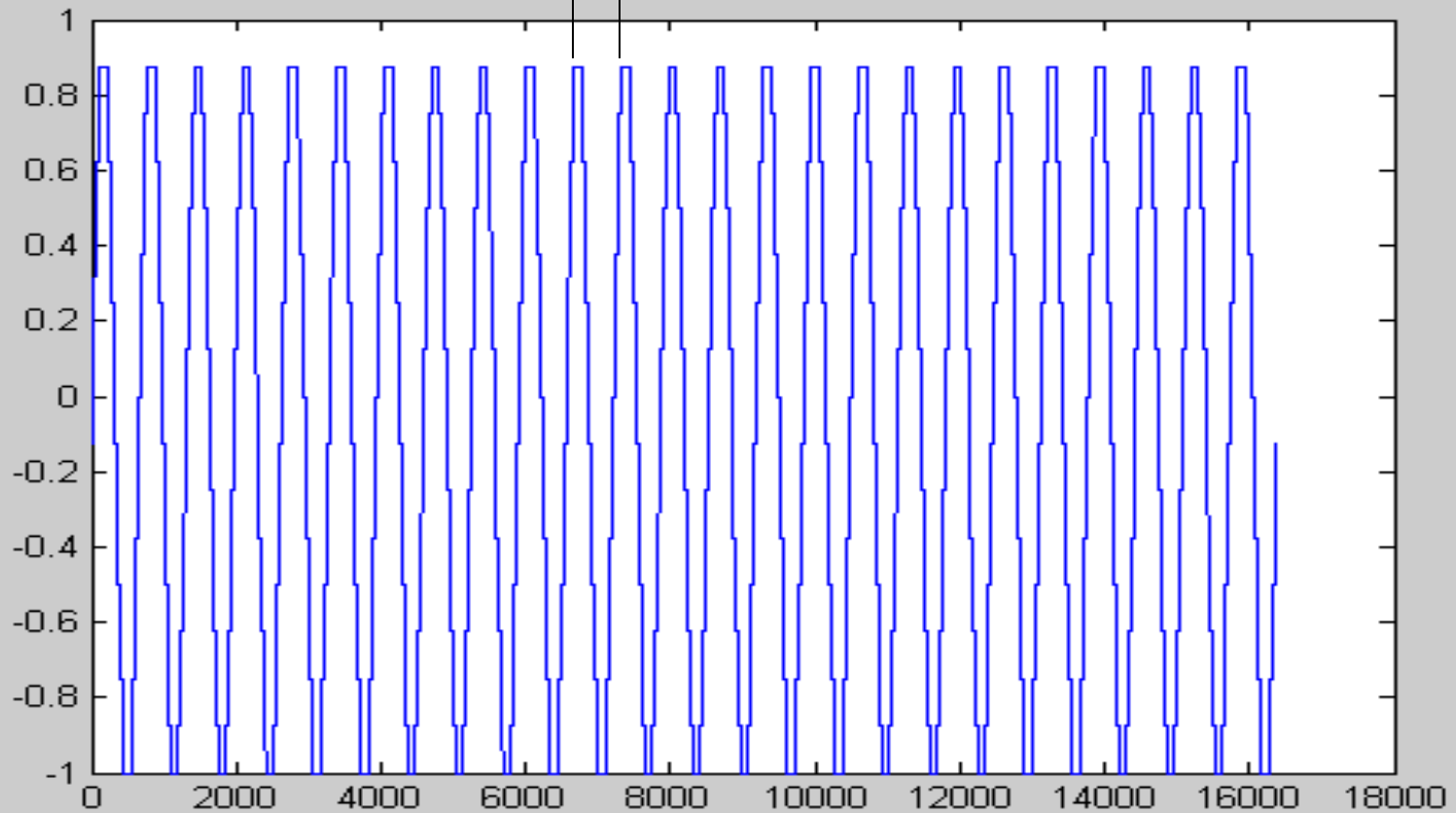
- Assume the effective clock rate (for either an ADC or a DAC) is arbitrarily fast
- Without Loss of Generality it will be assumed that $f_{\text{SIG}}=50\text{Hz}$
- Index on DFT will be listed in terms of frequency (rather than index number)

Matlab File: `afft_Quantization.m`

Quantization Effects

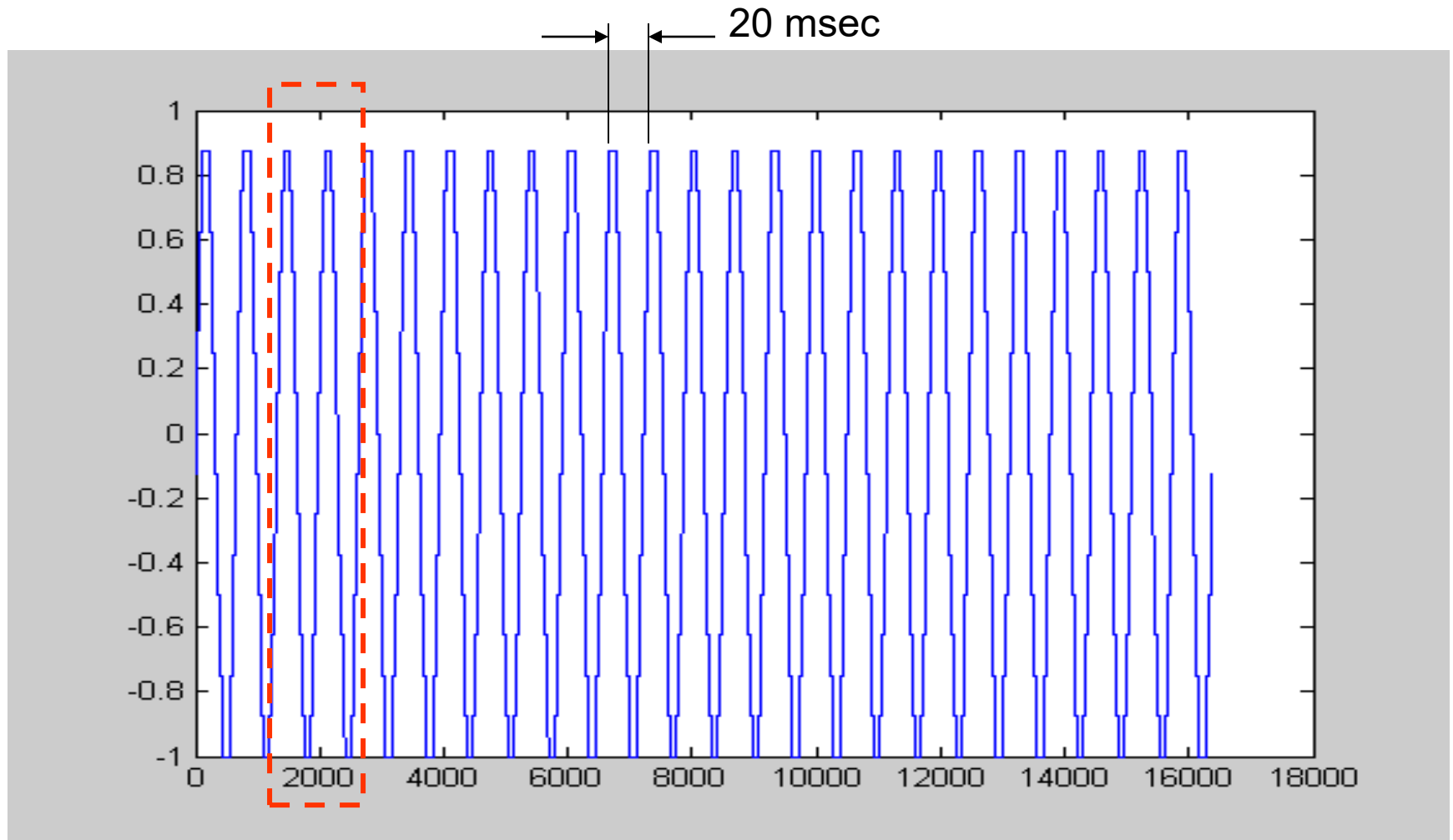
16,384 pts res = 4bits $N_p=25$

→ ← 20 msec



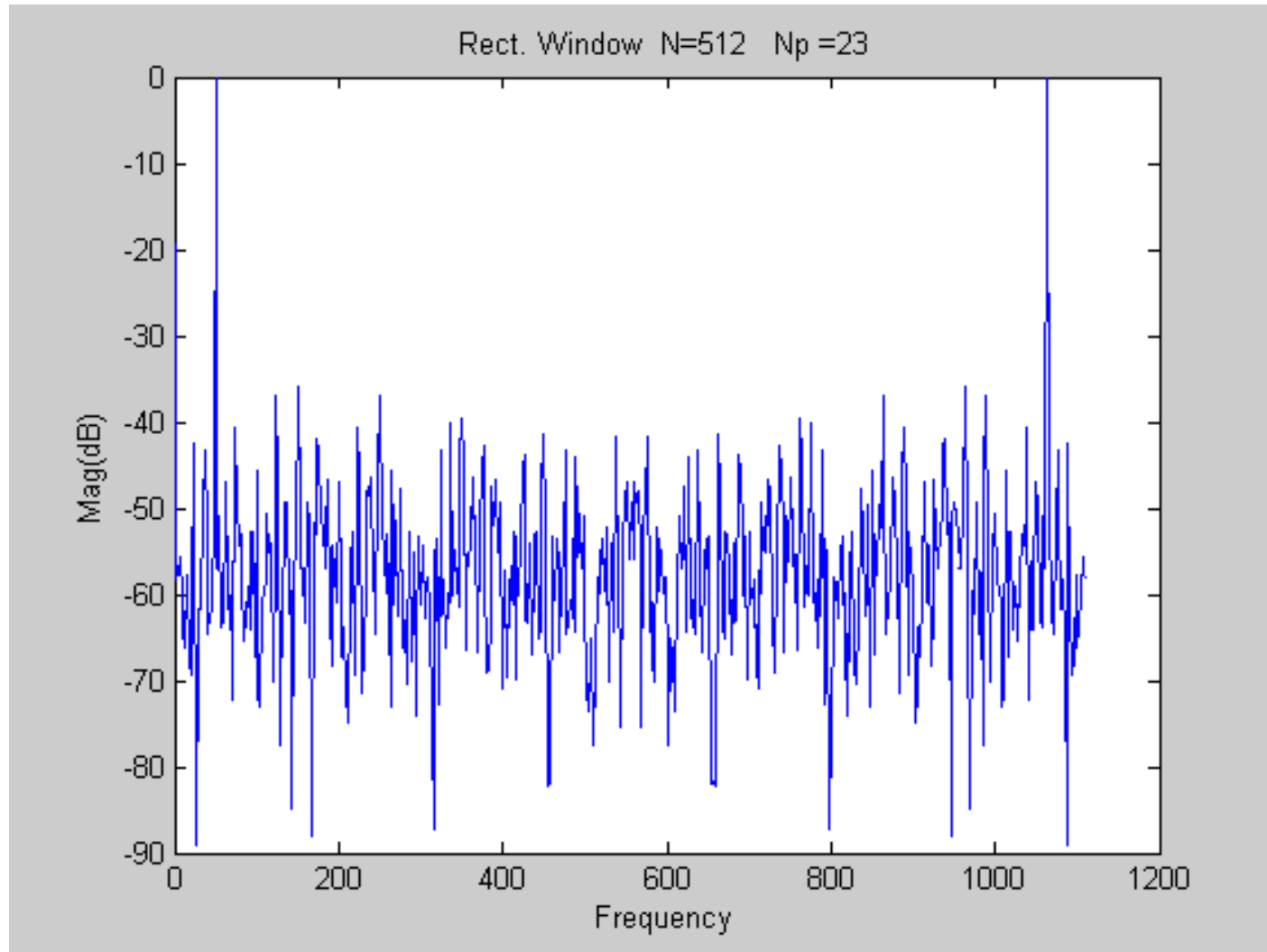
Quantization Effects

16,384 pts res = 4bits $N_p=25$



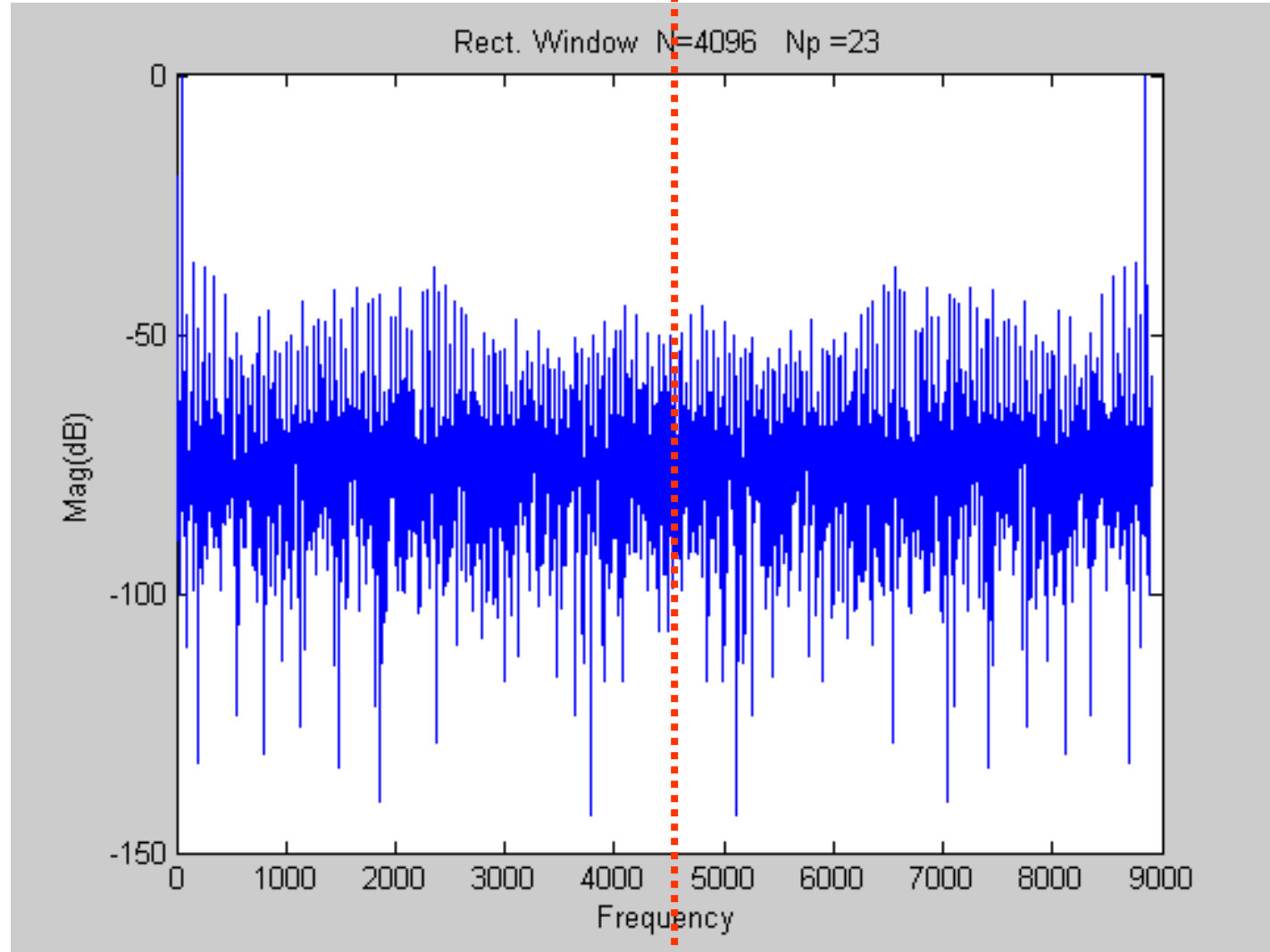
Quantization Effects

Res = 4 bits



Quantization Effects

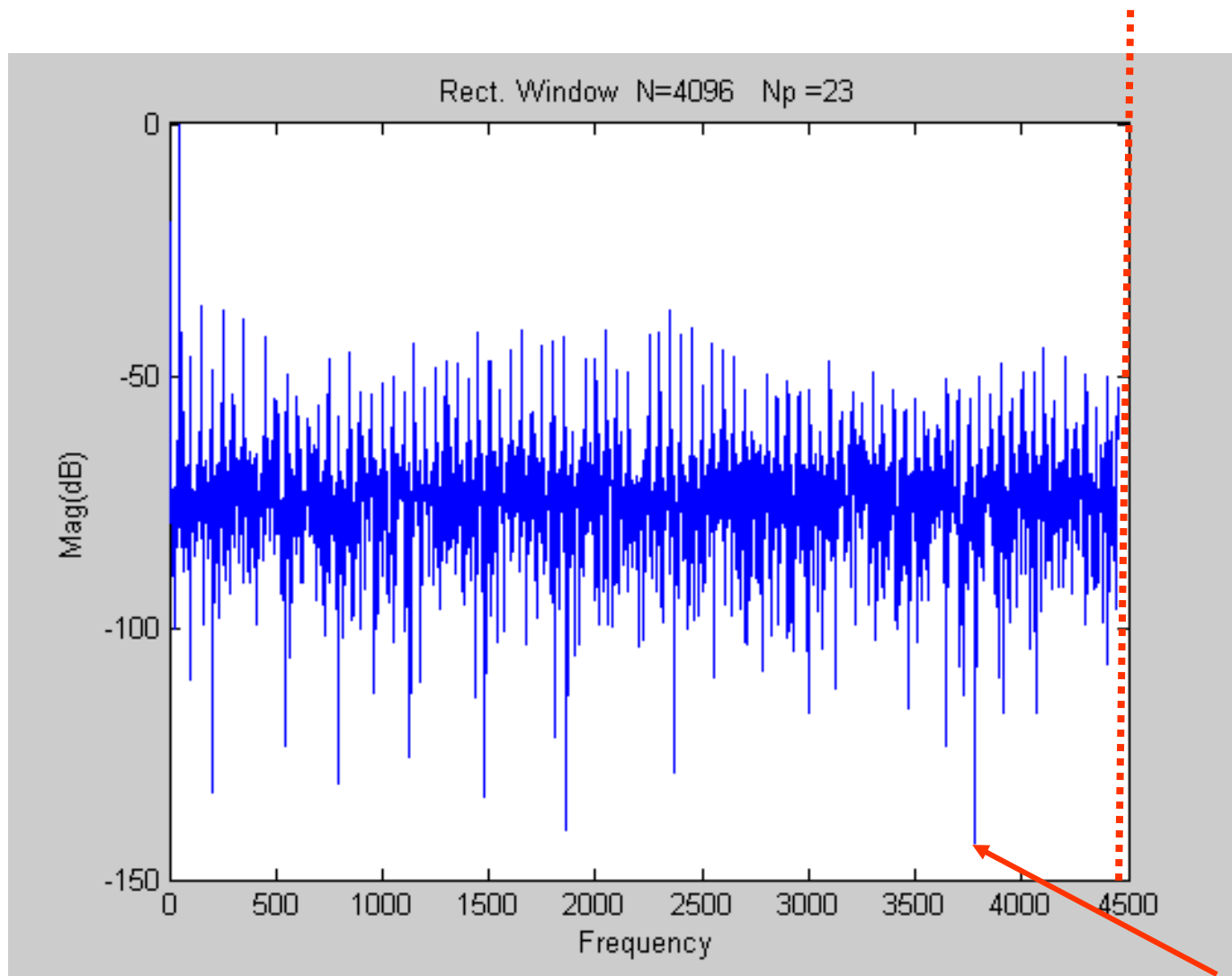
Res = 4 bits



Axis of Symmetry

Quantization Effects

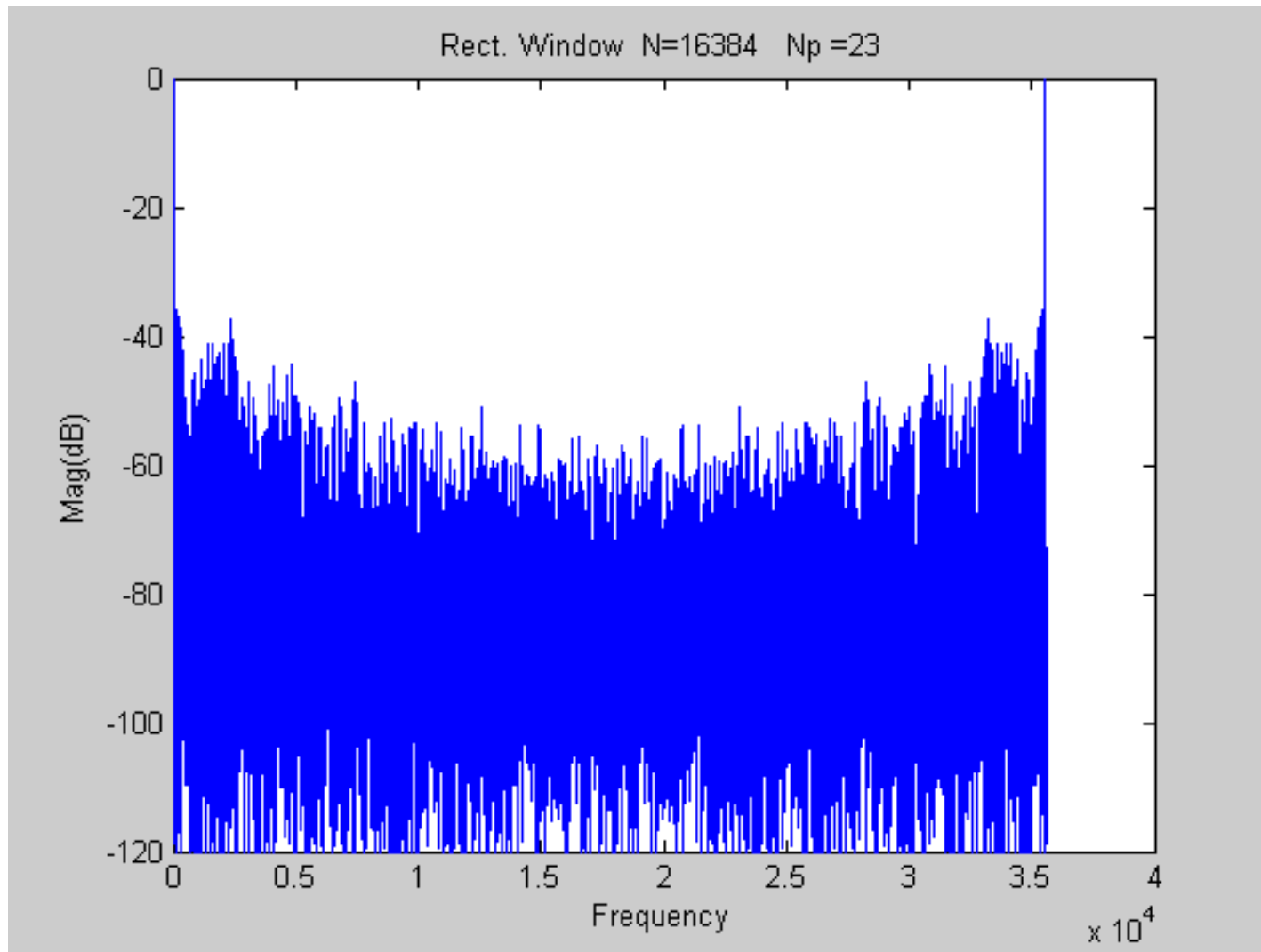
Res = 4 bits



Some components
very small

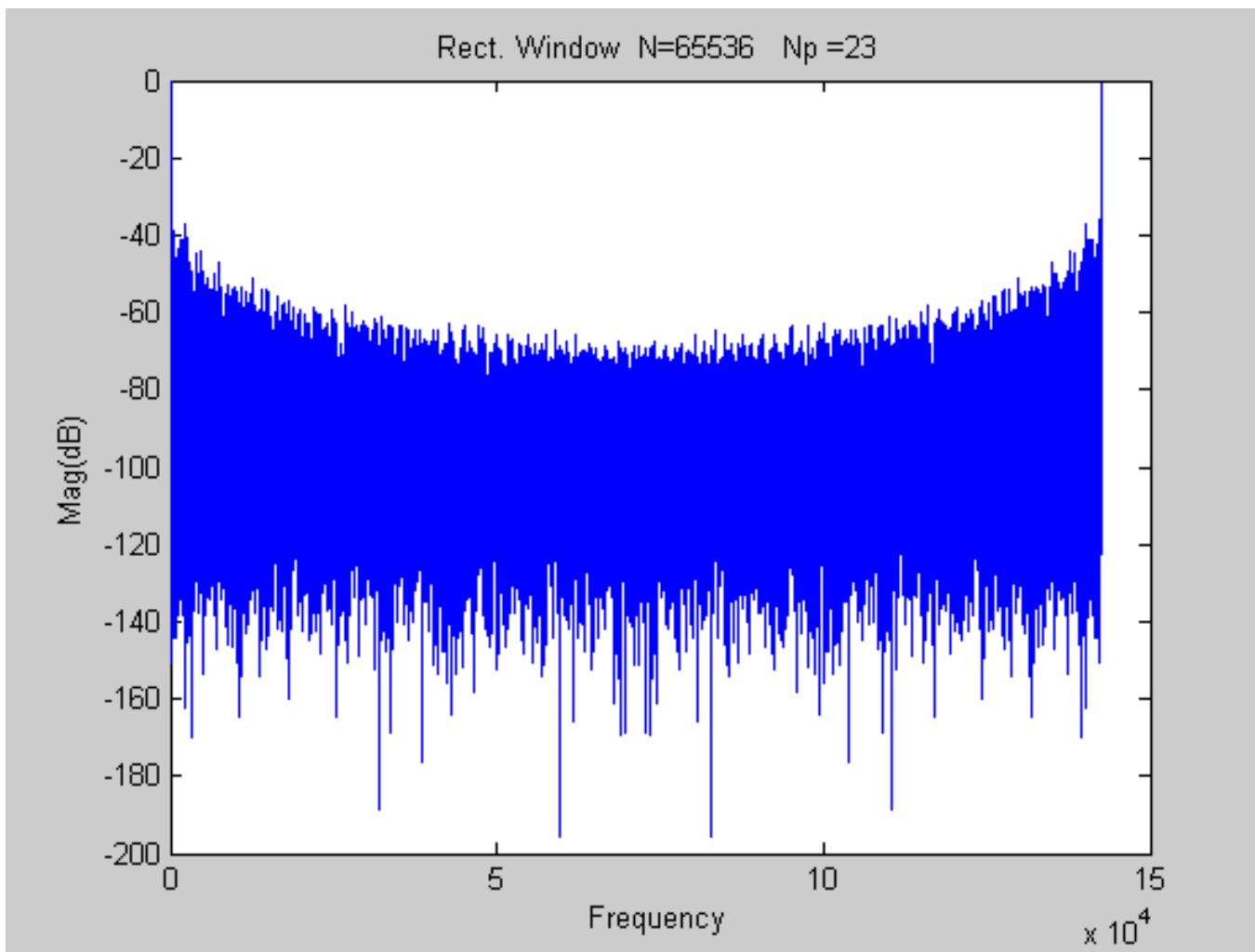
Quantization Effects

Res = 4 bits



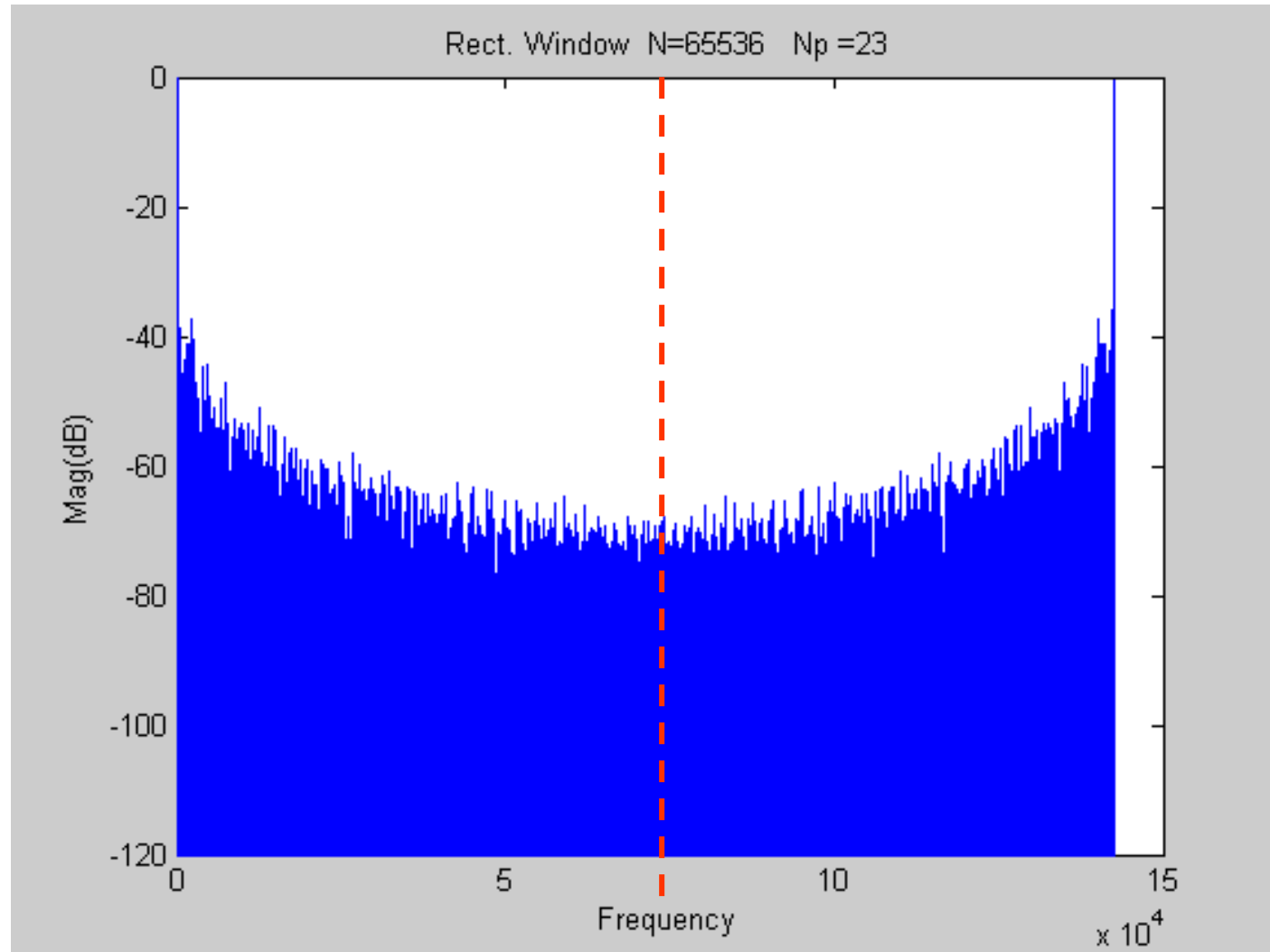
Quantization Effects

Res = 4 bits



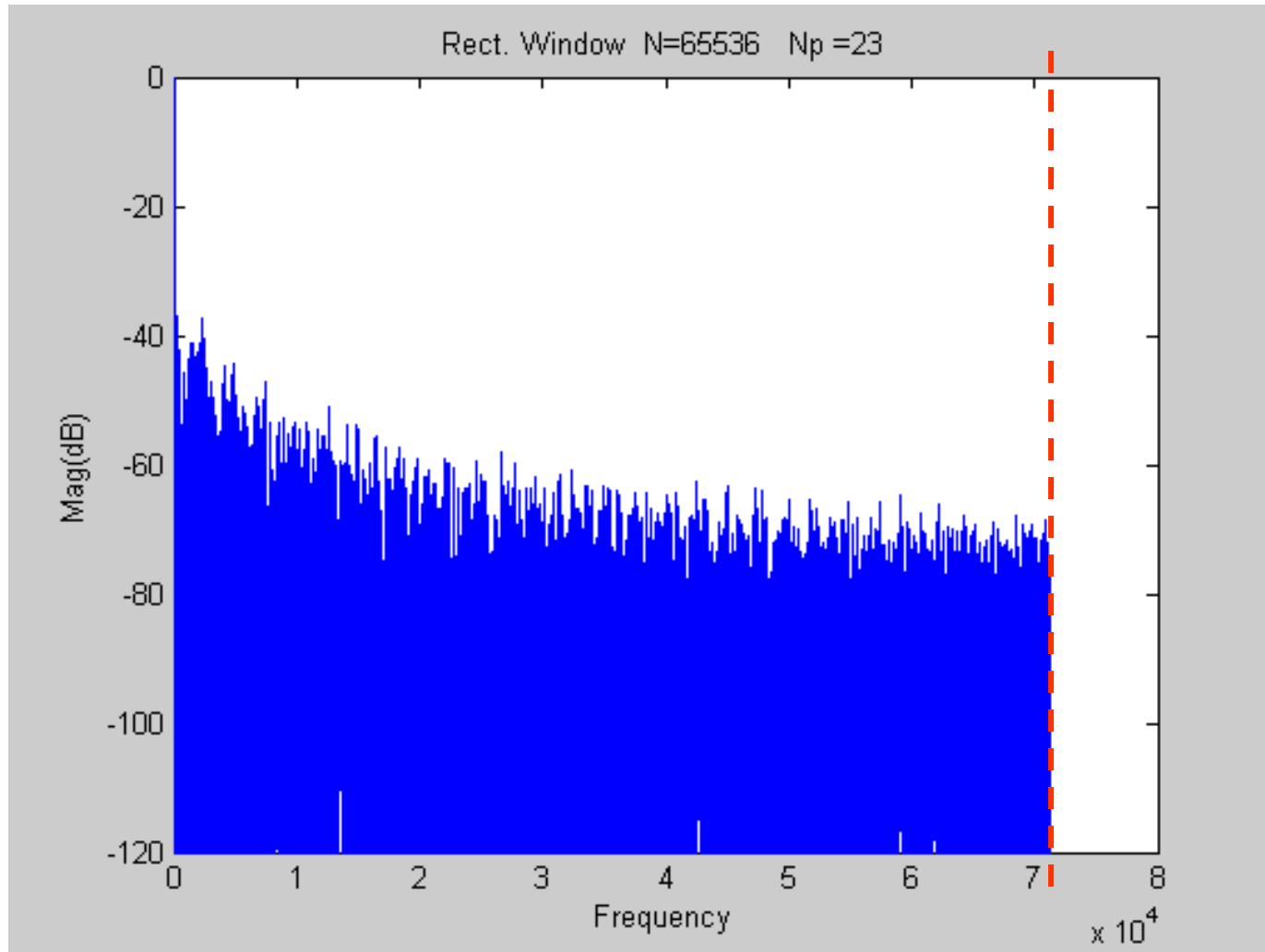
Quantization Effects

Res = 4 bits



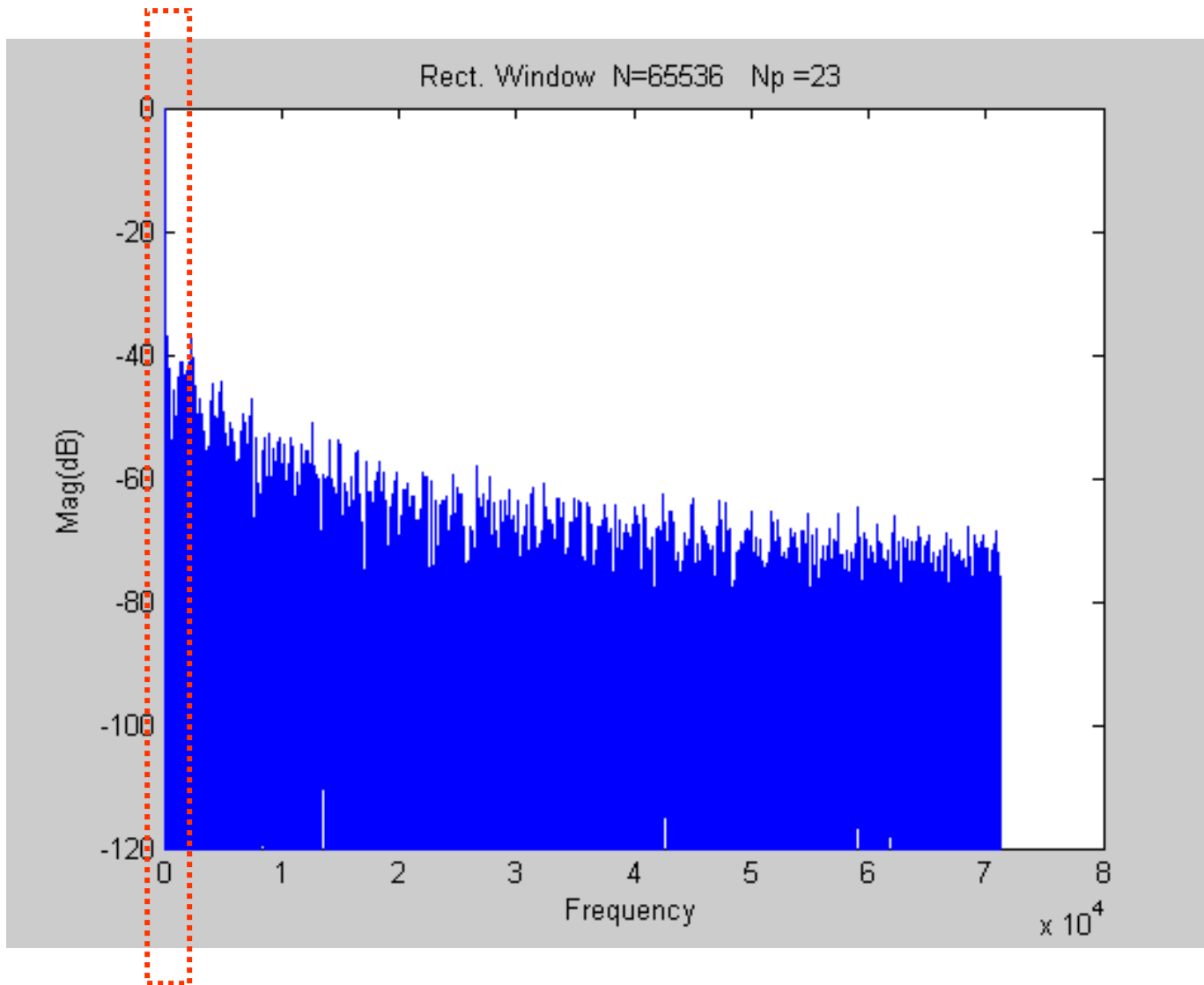
Quantization Effects

Res = 4 bits



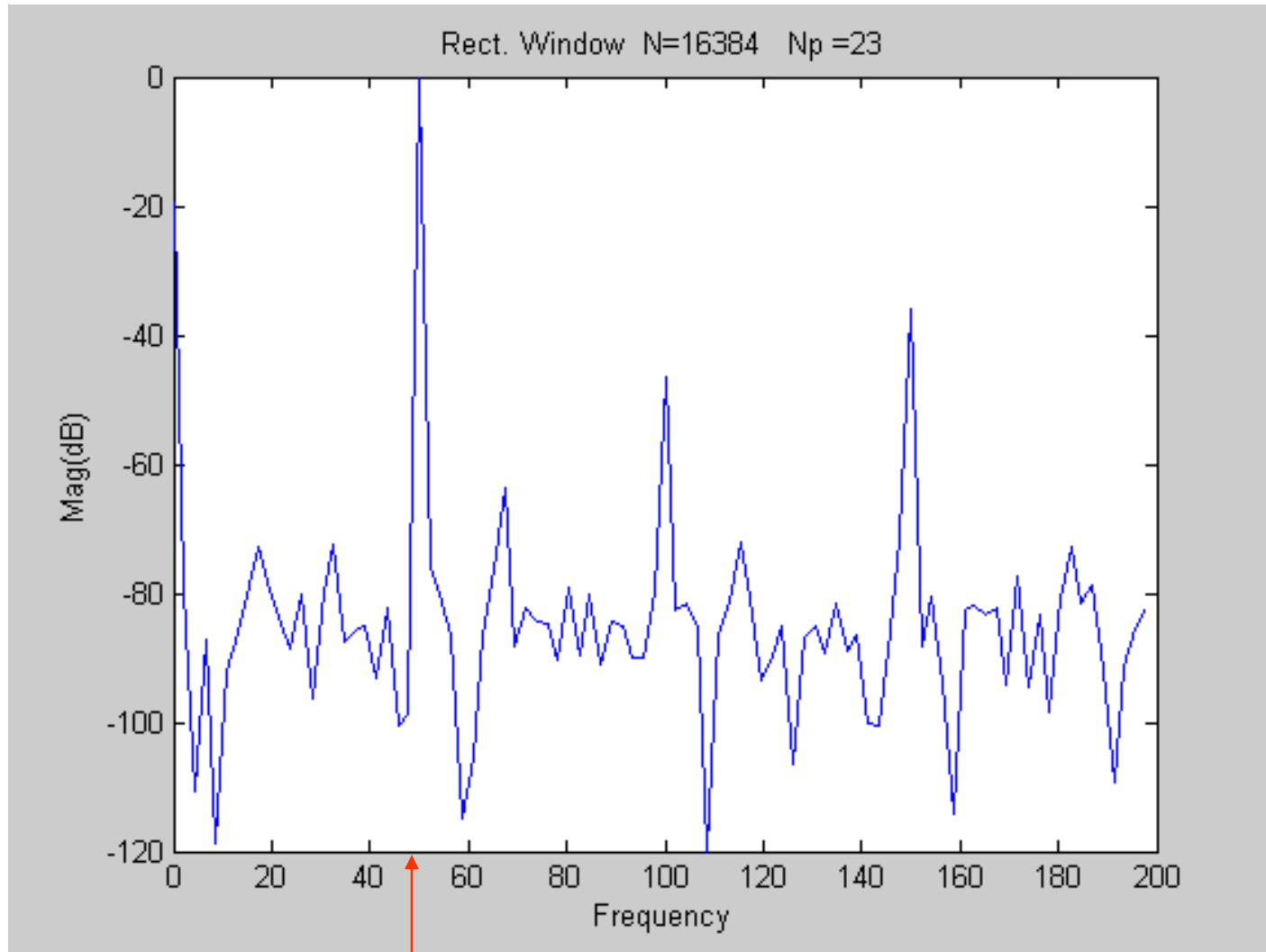
Quantization Effects

Res = 4 bits



Quantization Effects

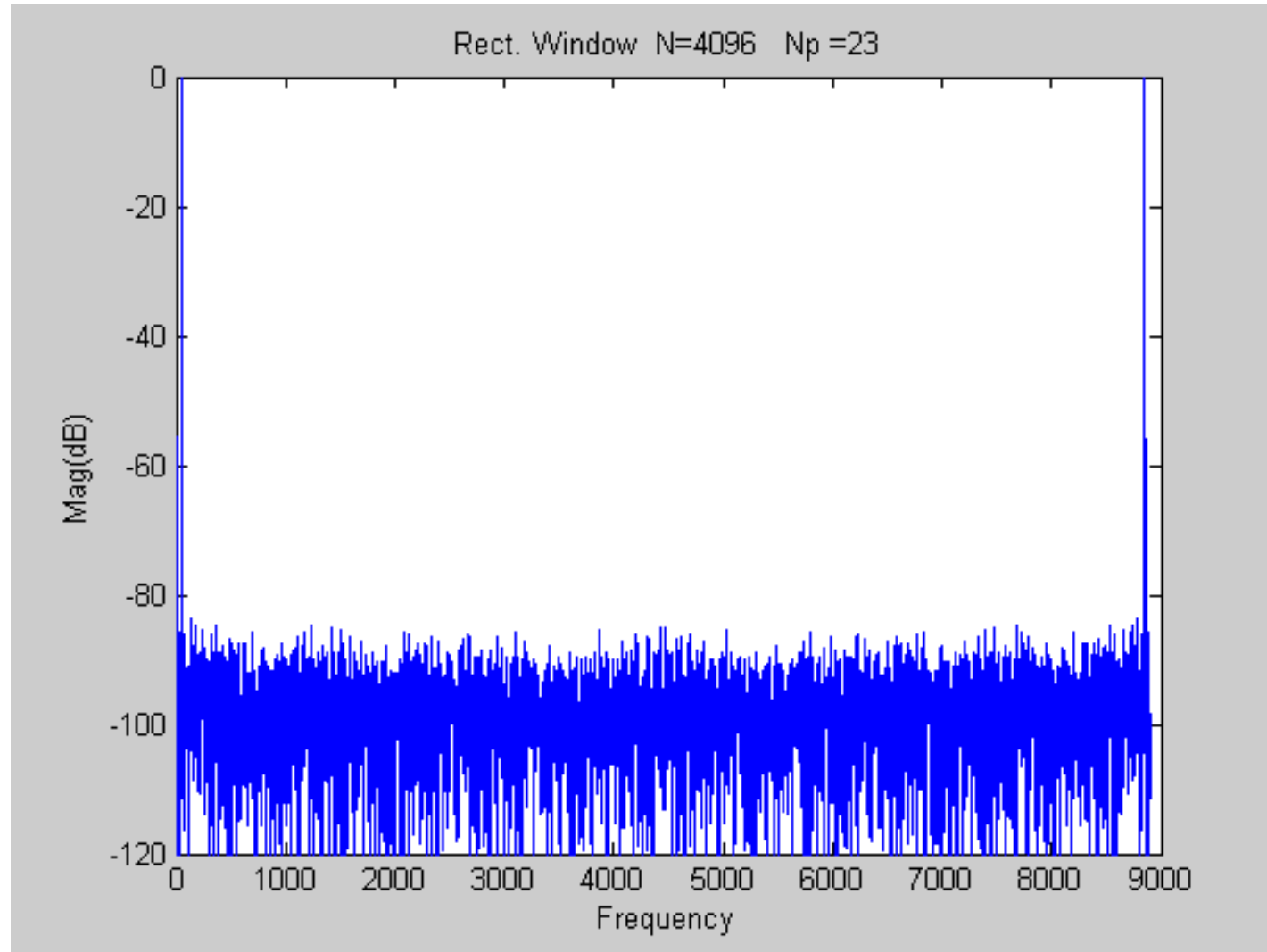
Res = 4 bits



Fundamental

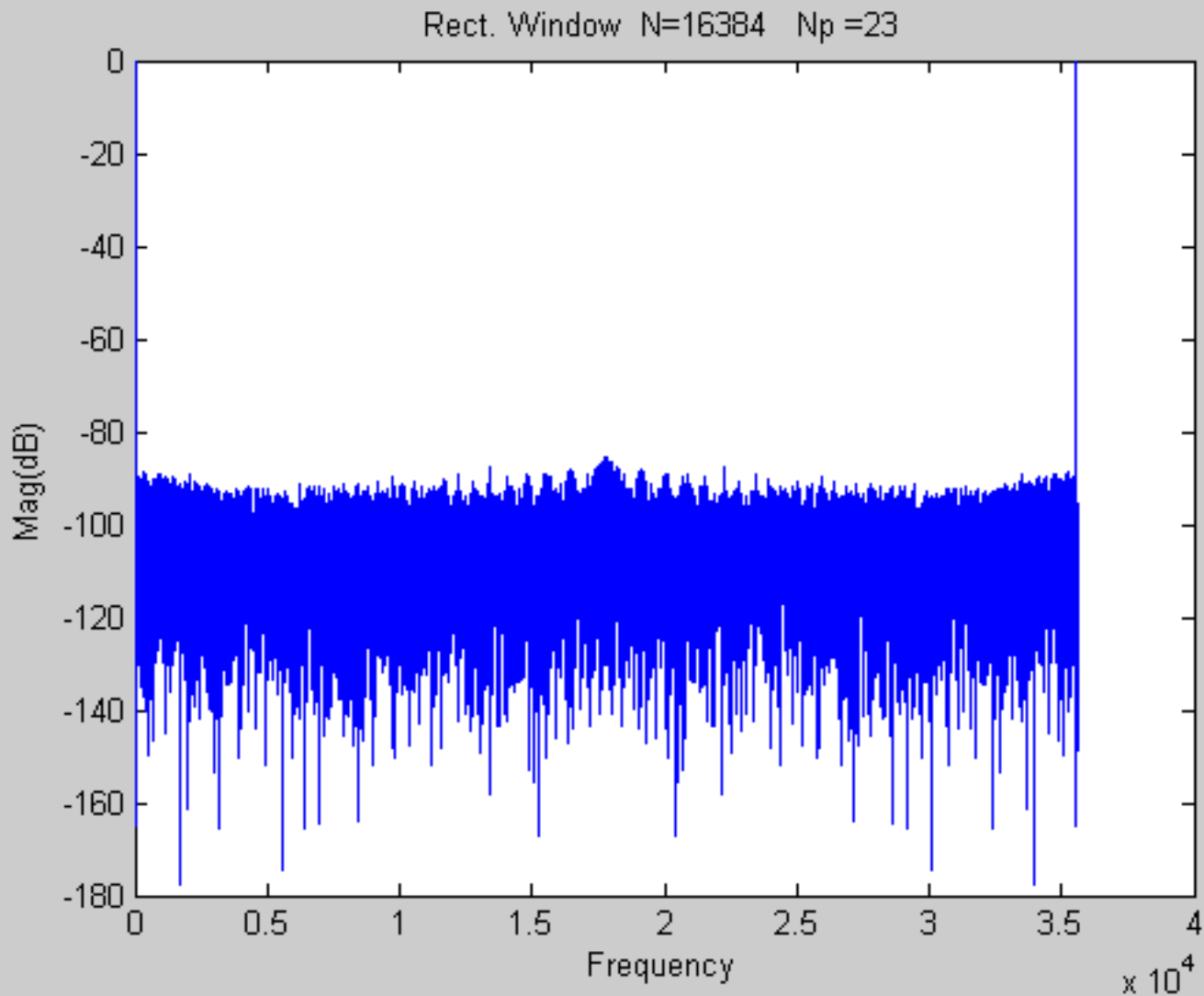
Quantization Effects

Res = 10 bits



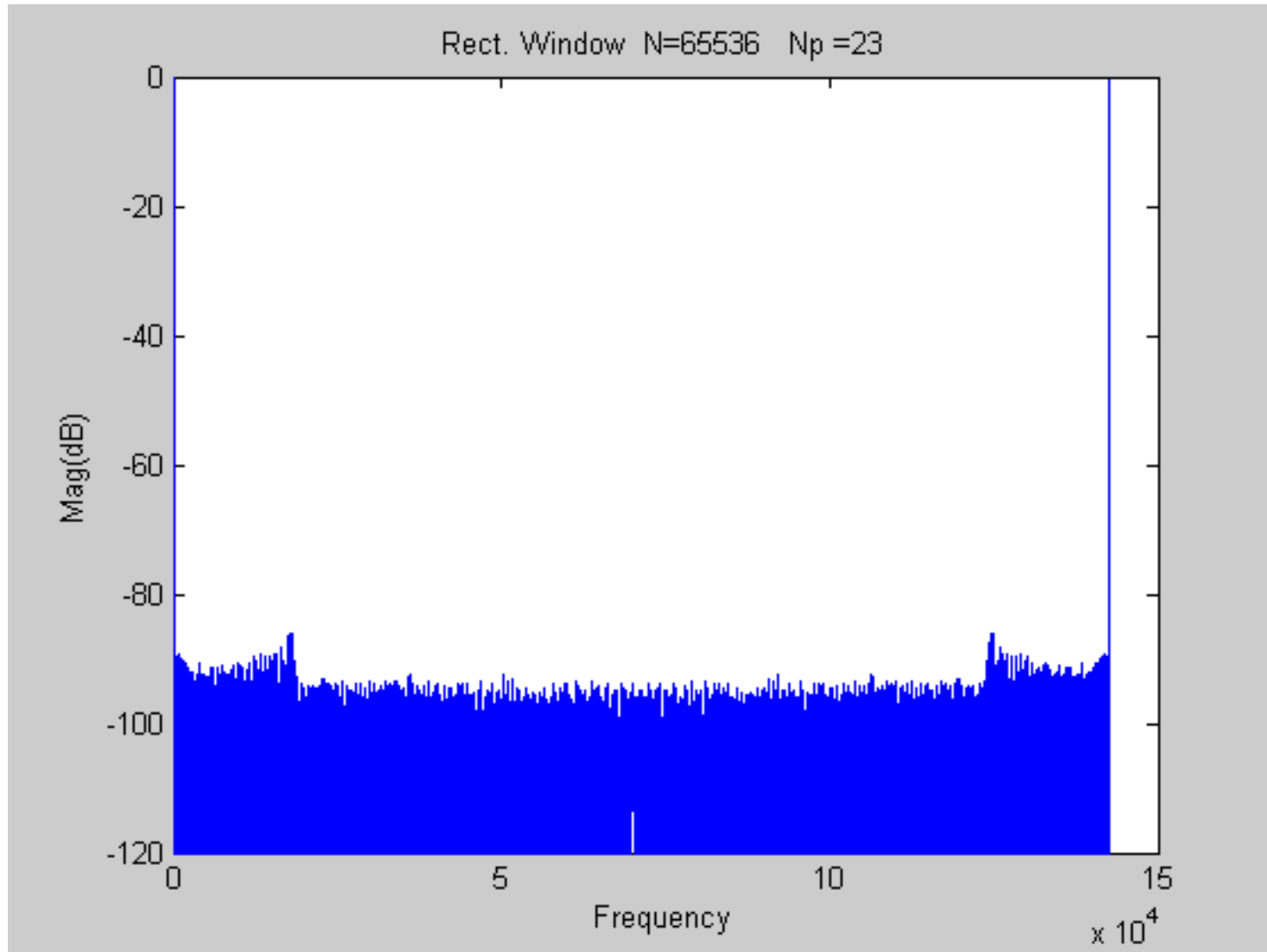
Quantization Effects

Res = 10 bits



Quantization Effects

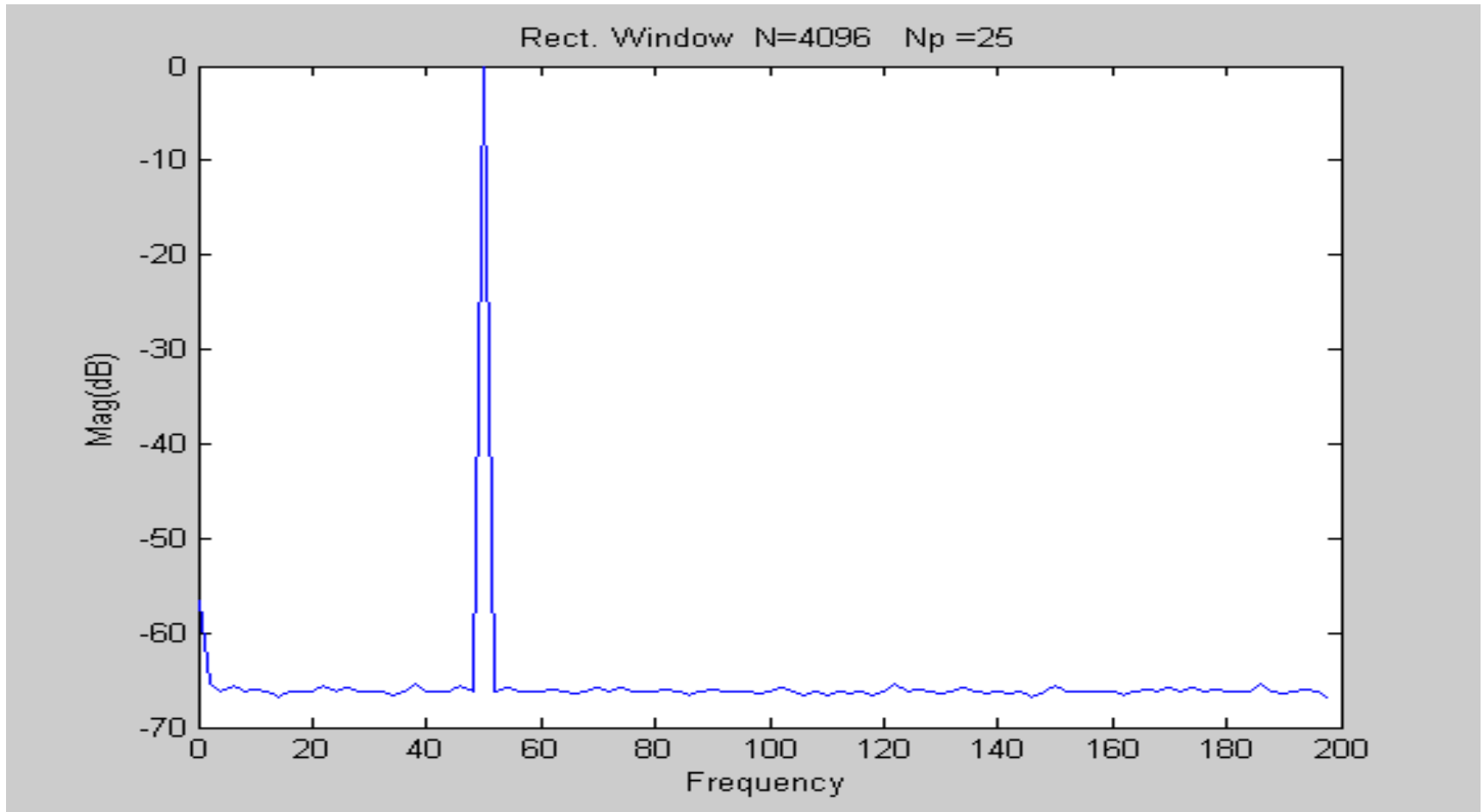
Res = 10 bits



Quantization Effects

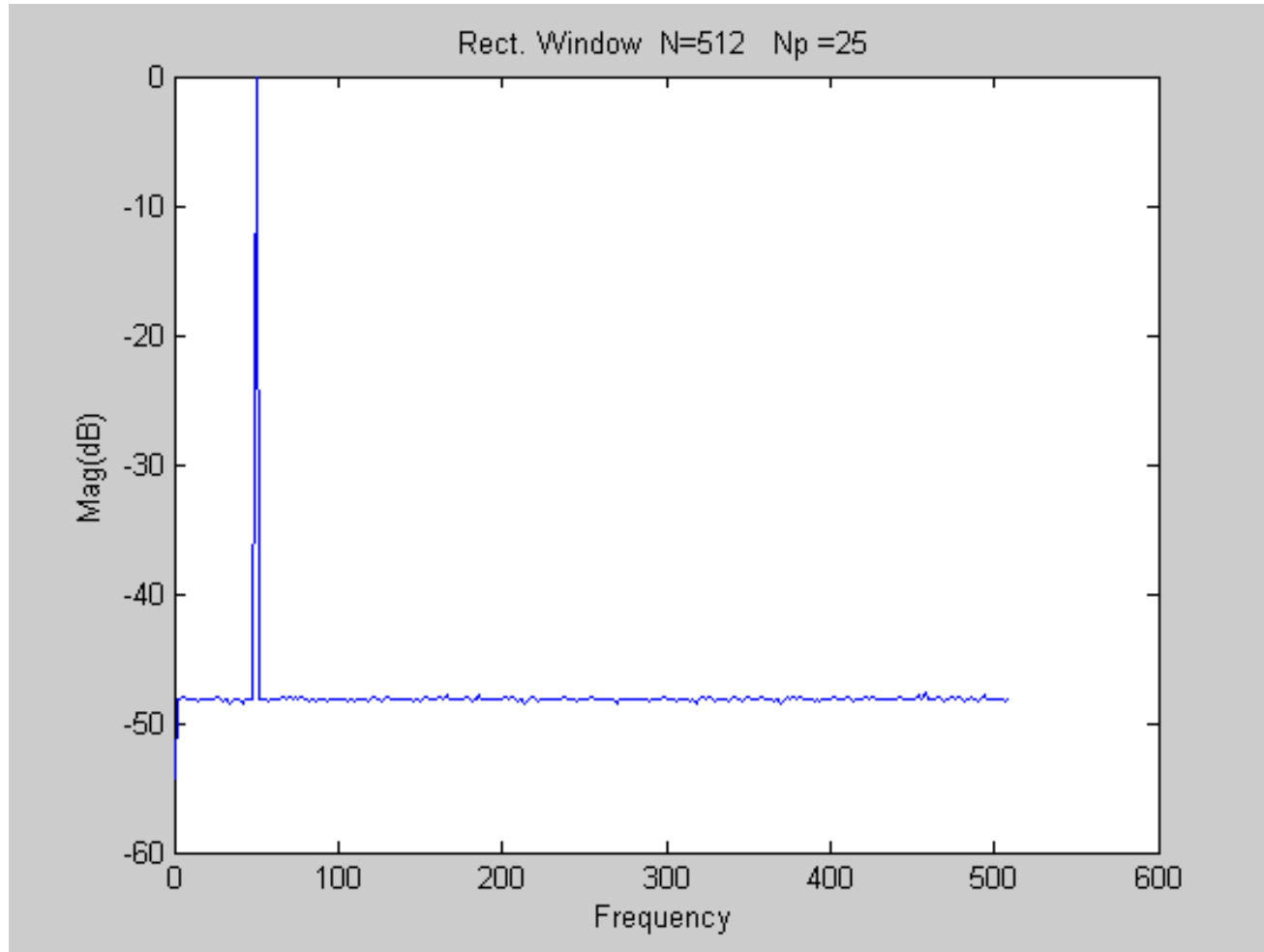
With $V_{in}=2v$ pp

Res = 10 bits



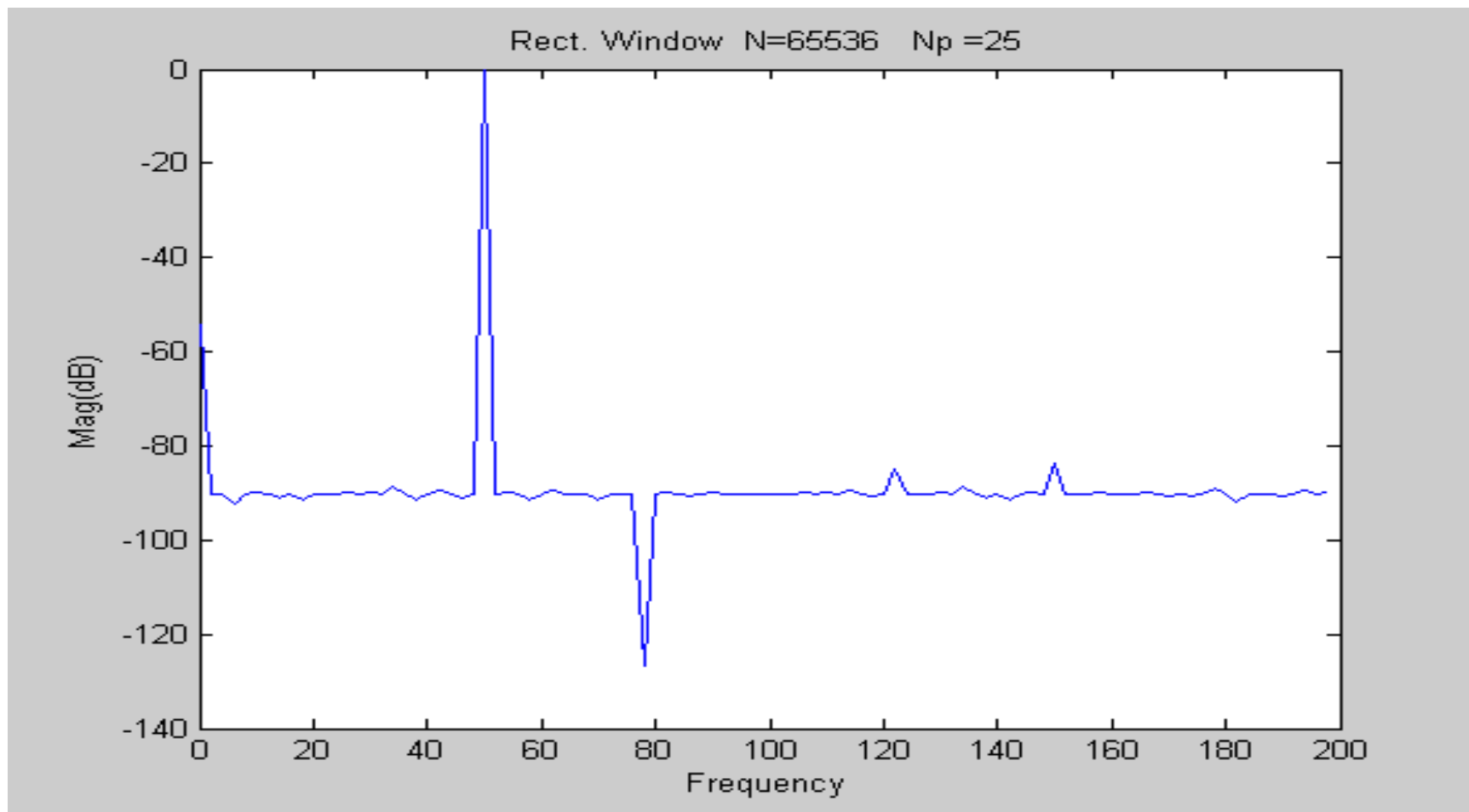
Quantization Effects

Res = 10 bits



Quantization Effects

Res = 10 bits



Summary of time and amplitude quantization assessment

Time and amplitude quantization do not introduce harmonic distortion

Time and amplitude quantization do increase the noise floor



Stay Safe and Stay Healthy !

End of Lecture 29